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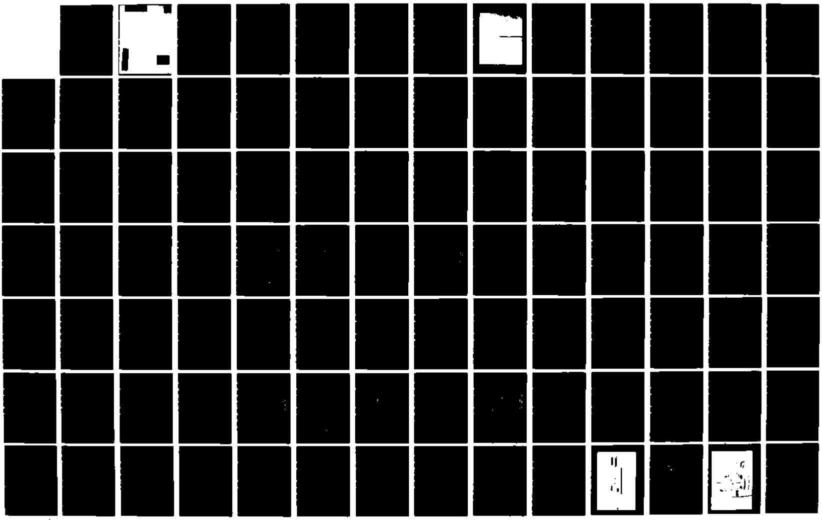

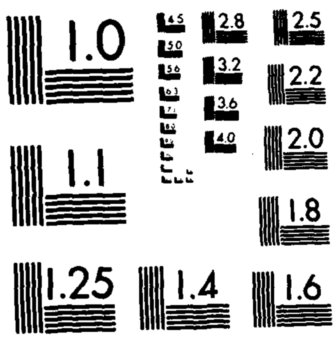

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ABSTRACT

A mathematical model of fatigue crack nucleation is described using the irreversible thermodynamics to quantify the damage caused by plastic straining. The model is based on the hypothesis that the entropy gain which results from dynamic irreversible plastic straining is a material constant. A random model of internal friction is used to calculate the irreversible part of the hysteresis energy dissipation rate, enabling the quantification of uncertainty through the variance of the dynamic plastic strain. In this

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A mathematical model of fatigue crack nucleation is described using the irreversible thermodynamics to quantify the damage caused by plastic straining. The model is based on the hypothesis that the entropy gain which results from dynamic irreversible plastic straining is a material constant. A random model of internal friction is used to calculate the irreversible part of the hysteresis energy dissipation rate, enabling the quantification of uncertainty through the variance of the dynamic plastic strain. In this manner, the

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20. Abstract continued

mathematical model for fatigue crack nucleation provides an estimate for longevity including confidence intervals under arbitrary temperature, environment, and loading history. The model explains the existence of an endurance limit. Although this analysis was conducted for an uncracked material leading to time estimates for crack nucleation, the theory can easily be extended to the case of crack propogation using the methods of fracture mechanics.

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SUMMARY OF IMPORTANT RESULTS

Structural fatigue fracture has thus far defied a comprehensive engineering solution. Although there have been several attempts at explaining the fundamental mechanism of material fatigue, none has proven completely successful as yet. In its broadest definition, fatigue is the gradual degradation of strength during repeated loading and unloading. Since the nature of this strength degradation is unknown, it is not possible to predict with confidence the structural strength at some arbitrary point in the service life. This leads to a "sudden death" nature of fatigue fracture: a structure which passes all inspections may fail unexpectedly, resulting in catastrophic loss of life and resources. This report describes a mathematical model of fatigue damage.

Fatigue fracture is known to consist of two parts: crack initiation whereby cracks form within the material and crack propogation whereby cracks grow until the structure fails. A theoretical basis for the nucleation of fatigue cracks has been proposed, resulting in a mathematical model of the fatigue mechanism. A new mathematical model for material damping has also been developed which is used in the calculation of the expected entropy rate resulting in a strain-level versus lifetime curve. Both models are consistent with laboratory measurements. The fatigue crack nucleation model under investigation here is based on diffusion mechanics, and was also used to calculate the approximate temperature rise for fatigue testing in vacuum. That analytical temperature rise also compares favorably with laboratory data. This model also can describe the endurance limit. If the probability distribution function is truncated at some low strain level, then strains below that level will be elastic; the plastic part of such a low strain level would be zero, yielding an infinite fatigue life. A clear result is that additional support for the validity of the critical entropy threshold of fracture has been presented. A

mathematical model of fatigue is an important and significant contribution, not only because it reproduces experimental results for fatigue life estimates including confidence intervals, but also because it has the capability of describing the influence of temperature and loading history on fatigue.

This mathematical model of fatigue is a general one describing the breakage of molecular bonds leading to strength degradation. Figure 1 shows a typical crack which always forms at the base of the cantilever beam fatigue specimens after about a two percent decrease in natural frequency. In this study, crack nucleation is viewed as a random process whereby a microscopic crack suddenly appears in the material whenever the local fracture entropy threshold has been exceeded. This sudden appearance of a crack would be described mathematically by a delta function. However, the propogation of the crack would occur from the same phenomenon; as the local fracture entropy threshold is exceeded, the crack would extend due to the local breakage of molecular bonds within the material. The analysis of crack propogation by the entropy threshold is being initiated at this writing.

The development of a mathematical model accurately describing the physical phenomenon of fatigue is a significant advance. Once the validity of this model is established, the application of computer-aided design for maximum longevity is possible. Such a design procedure might provide more efficient engineering design and developement, testing, and evaluation for new structures, as well as more accurate replacement criteria for maintenence. It is likely that an accurate mathematical model for fatigue including the effects of temperature, environment, and loading history could provide significant cost savings in structural design, testing, and maintenance.

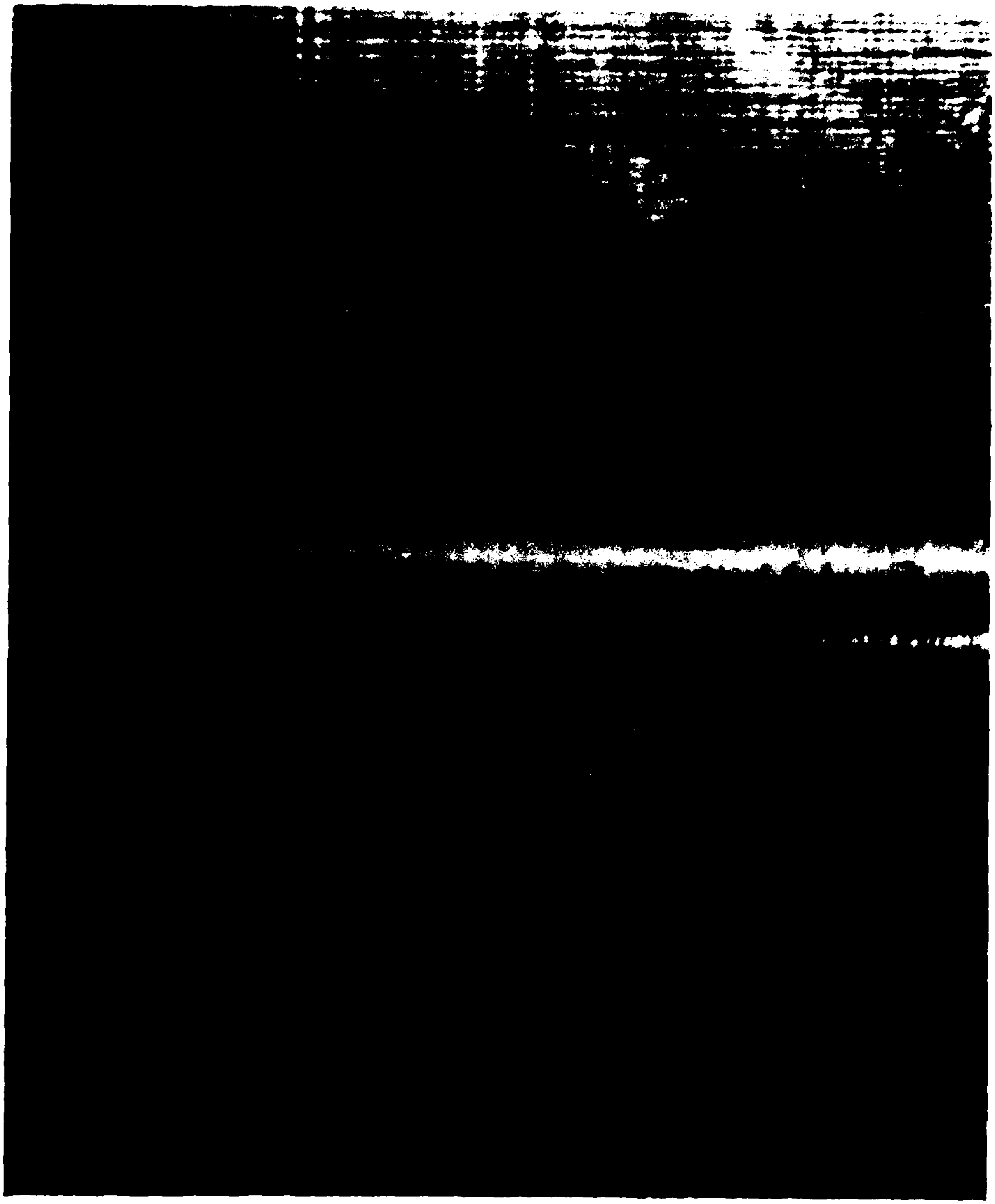

Figure 1. Photograph of Crack Which Just Forms at the Base of the Base-Excited Cantileaver Beam After About a 2% Decrease of Natural Frequency.

LIST OF PUBLICATIONS AND PRESENTATIONS
1 OCT. 1982 - 1 OCT. 1983

1. Whaley, P. W., "A Thermodynamic Approach to Material Fatigue," Proceedings, ASME International Conference on Advances in Life Prediction Methods, Albany, New York, 18-20 April, 1983, p 41.

2. Whaley, P. W., Y. C. Pao, and K. N. Lin, "Numerical Simulation of Material Fatigue by a Thermodynamic Approach," Proceedings, 24th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, Lake Tahoe, Nevada, 2-4 May 1983, p 544.

3. Whaley, P. W., P. S. Chen, and G. M. Smith, "Continuous Measurement of Material Damping During Fatigue Tests," Submitted for publication to Experimental Mechanics, July 1983.

4. Whaley, P. W., "The Critical Entropy Threshold: A Mathematical Model for Fatigue Crack Nucleation," Submitted for Publication to the International Journal of Engineering Science, October, 1983.

5. Whaley, P. W., "A Stochastic Model for Material Damping Based on Random Yielding," Submitted for Publication to the International Journal of Engineering Science, October, 1983.

LIST OF FIGURES

NOMENCLATURE

ENGLISH:

D_i	irreversible part of dissipated energy
E	Young's modulus
E_s	storage modulus normalized to Young's modulus
J	function used in selecting parameters σ_o, ε_m by gradient search techniques
C_σ	ratio of calculated stress to ultimate stress
C_ε	arbitrary constant of proportionality between loss factor variance and plastic strain variance
$P(\varepsilon)$	random yielding probability distribution function
S	local entropy gain
B	tip amplitude of vibration
d	diameter of tip accelerometer
e	distance from the tip accelerometer top to the beam neutral axis
h	beam thickness
I	area moment of inertia of beam cross-section
J_G	mass moment of inertia of added accelerometer
k_i	eigenvalue of the beam corresponding to the i^{th} natural frequency; $\lambda_i=k_i\ell$
ℓ	beam length
m	mass of the added accelerometer
$q_i(t)$	generalized time-response function corresponding to the ith natural frequency
R_i	eigenfunction constant
x	position along the beam
$y(x,t)$	beam transverse displacement
Z	input displacement of the beam base

Greek:

β	parameter describing the change of material strength with time under dynamic loading
ε	strain
ε_f	maximum strain
ε_m	mean yield strain
ε_s	dynamic strain amplitude
γ	frequency exponent describing frequency dependence of elastic loss factor
η_e	elastic part of loss factor
η_o	reference elastic loss factor
η_p	plastic part of loss factor
$\eta_s(\varepsilon)$	total frequency-and amplitude-dependent loss factor
$\eta_s{}^*$	total loss factor predicted by the model for random yield at a strain near the yield point
θ	temperature; also maximum angular vibration amplitude of the beam tip
ν_s	variance of plastic strain
σ_s	dynamic stress amplitude
σ_u	ultimate static stress
$\sigma_u{}^*$	maximum static stress predicted by the model for random yielding
ϕ_e	phase between stress and strain from elastic loss factor
ϕ_p	phase between stress and strain from plastic loss factor
ω	frequency, radians/sec
$\phi_i(x)$	i^{th} mode shape function
ϕ'_i	$d\phi_i/dx$
ϕ''_i	$d^2\phi_i/dx^2$

ω_i — i^{th} natural frequency

μ — mass ratio: accelerometer to beam
$\mu = m/\rho A \ell$

ν_1 — radius of gyration ratio: accelerometer to beam length
$J_G = m\kappa^2 \ell^2$; κ = accelerometer radius of gyration

ν_2 — accelerometer height ratio
$\nu_2 = e/2\ell$

ν_3 — accelerometer diameter ratio
$\nu_3 = d/2\ell$

ρ — beam density

THE CRITICAL ENTROPY THRESHOLD: A MATHEMATICAL MODEL FOR FATIGUE CRACK NUCLEATION

by

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ABSTRACT

A mathematical model of fatigue crack nucleation is described using the irreversible thermodynamics to quantify the damage caused by plastic straining. The model is based on the hypothesis that the entropy gain which results from dynamic irreversible plastic straining is a material constant. A random model of internal friction is used to calculate the irreversible part of the hysteresis energy dissipation rate, enabling the quantification of uncertainty through the variance of the dynamic plastic strain. In this manner, the mathematical model for fatigue crack nucleation provides an estimate for longevity including confidence intervals under arbitrary temperature, environment, and loading history. The model explains the existence of an endurance limit. Although this analysis was conducted for an uncracked material leading to time estimates for crack nucleation, the theory can easily be extended to the case of crack propogation using the methods of fracture mechanics.

INTRODUCTION

Structural fatigue fracture has thus far defied a comprehensive engineering solution. Although there have been several attempts at explaining the fundamental mechanism of material fatigue, none has proven completely successful as yet. In its broadest definition, fatigue is the gradual degradation of strength during repeated loading and unloading. Since the precise nature of this strength degradation is unknown, it is not possible to predict with confidence the structural strength at some arbitrary time in the service life. This leads to a "sudden death" nature of fatigue fracture: a structure which passes all engineering inspections may fail unexpectedly, resulting in catastrophic loss of life and resources. This paper describes the theory of a new mathematical model of fatigue which describes the fundamental mechanism of local fracture.

There have been at least three main approaches to the theoretical basis of fatigue: damage mechanisms, energy dissipation relationships, and dislocation dynamics. Miner[1] hypothesized that the empirical stress-cycles diagram in sinusoidal reversed loading provides the basis for the fatigue damage mechanism. In this view, the percent life consumed at various different stress levels must sum to one at failure. This hypothesis has not proven true in general as order and type of loading also influences fatigue life, although the Miner criterion can provide qualitative preliminary data.

Enomoto[2] was among the first to hypothesize that the energy dissipated above some non-damaging threshold is a constant. That hypothesis is also not true in general, as even the plastic part of dissipated hysteresis energy is known to increase with lifetime. Finally, there have been many approaches relating dislocation motion to fatigue resulting in the qualitative interpretation that fatigue cracks are caused by dislocation pileups[3].

The fatigue damage interpretation persists in the recent literature. Reference[4] describes an analytical approach based on the Miner criterion attempting to quantify structural fatigue life under specific loading conditions. Reference[5] offers the suggestion that the Palmgren-Miner rule is merely a special case of a cumulative damage theory and that such theories are useful for specifying qualitative guidelines. Reference[6] suggests a damage law for creep based on fracture mechanics including a damage parameter which has an ambiguous physical meaning. Reference[7] is a similar fracture mechanics interpretation where damage is considered to be proportional to the plastic zone size in the vicinity of a crack tip. Reference[8] describes a model for crack propogation where failure of an element results when damage has accumulated past a certain threshold value. The stored damage is considered to accumulate in the form of dislocation pileups. All of these damage approaches are physically reasonable as it is intuitively obvious that load cycling will gradually degrade strength in the form of some

permanent structural change. However, none of the damage approaches provides a physical definition or quantitative description for this damage quantity.

Reference[7] identifies damage as somehow related to plastic strain through the plastic zone near the crack tip. This leads to the evaluation that fatigue occurs as a result of irreversible energy dissipated within the material, specifically the plastic part of the energy. This recent work is quite similar to Enomoto's[2] hypothesis that failure occurs when the plastic part of the dissipated energy accumulates past a certain amount, but that plastic strain energy threshold still has not been quantified. Feltner and Morrow[9] conducted experiments on steel with favorable agreement to such a plastic strain energy hypothesis, but Martin[10] concluded that this plastic hysteresis energy hypothesis is not true in general. Later, Halford[11] modified this hypothesis to allow a variable plastic hysteresis energy threshold with fatigue life.

Fatigue fracture is known to consist of two parts: crack initiation whereby cracks form within the material and crack propogation whereby cracks grow until the structure fails. Since no failure would be expected to occur unless cracks are present, the conservative approach is to evaluate the time to propogate cracks. In the literature, there has been an emphasis on crack propogation over the past several years. References[12]-[16] are energy-based approaches to the growth of cracks. Reference[12] postulates that the rate

of crack propogation is a function of the strain energy density factor, representing another energy based approach to fatigue fracture. Reference[13] is an even more basic approach to the energy transfer during fracture, extending the Griffith fracture theory to cyclic loading using the Gibbs free energy of solids. The irreversible thermodynamics is used to derive a fatigue crack propogation rate dependent on the plastic strain.

The plastic strain energy is recognized as being intimately related to fatigue damage. In a technical note, Bodner,et.al.[14] suggest that the Paris crack growth law may be a special case of a more general relationship which would describe crack growth rate as proportional to the plastic work rate. The presence of plastic deformation is known to be essential for fatigue failures, and can be related to the experimental determination of a fatigue limit through damping tests[15]. Energy balance equations have also been used to model creep deformation and for linear elastic materials the well known J-integral can be derived[16]. Reference[17] is a related investigation based on molecular bonding considerations.

With regard to energy relationships in fatigue, there have been relatively few thermodynamic treatments including the irreversible nature of plastic strain and the fact that temperature is a dependent variable. This may be due to the fact that irreversible thermodynamics is a more recent area of investigation. References[18] and[19] describe a theory

of irreversible thermodynamics of solids. Eringen[18] derives equations for irreversible work and entropy of solids, and cites large deformation of elastic solids accompanied by small irreversible changes as an example. This is a good working definition of fatigue and provides encouragement for analysis of irreversible thermodynamics of fatigue. In Reference[19], a mathematical approach is described for thermodynamic analysis of elastic solids.

Any irreversible thermodynamic analysis of fatigue must include a procedure for calculating entropy as related to the plastic strain energy. Reference[20] describes this relationship in terms of the yield state, elastic state, and strain history. A transition region is defined for moving from elastic state to yield state. References[3] and[21] include a description of plastic strain through an analysis of the mechanics of dislocations. Reference[3] asserts that plastic deformation arises from the movement and generation of dislocations. Equations for entropy production in terms of plastic deformation, heat flux and strain energy are given. Reference[21] analyzes flow in solids and therefore interprets plastic strain as the primary variable. Thermodynamic equations are derived and plastic work is considered to be converted to heat and expended in changing the defect structure of the solid. Both References[3] and[21] contain the implicit assumption that an entropy gain accompanies plastic strain and entropy rate is positive

definite. Reference[22] is a similar analysis of thermodynamics of dislocations including equations for entropy and plastic strain.

The practical area of variable loading in fatigue has only received moderate treatment in the literature. Although there have been a few studies of fatigue under random excitation, no general relationship exists at this time. References[23] and[24] are analytical approaches to fatigue under random excitation using the Palmgren-Miner cumulative damage approach. Reference[25] is a similar stochastic approach to fatigue including deformation energy. Reference[26] is a more general approach emphasizing the reliability of the structure. References[27]-[30] are descriptions of a cumulative damage model using a Markov vector and defining a duty cycle. Random excitation can be described in the definition of the duty cycle. Reference[31] is an analysis of random crack growth predicted by various approaches, arguing that probabilistic models are a more rational method of accounting for variability.

This paper presents a mathematical model for fatigue crack nucleation using an irreversible thermodynamic analysis of dynamic loading. The material is assumed to be initially free of cracks and a basis for the appearance of local cracks is presented. The significance of this is that a theoretical basis for the fatigue mechanism will allow modeling and analysis of arbitrary loading and environment, including the effects of corrosion and temperature. In this paper, the theoretical basis for this hypothesis is presented along with comparisons of model predictions with experimental results.

THE CRITICAL ENTROPY THRESHOLD OF FRACTURE

It is hypothesized that the entropy gain at local fracture is a material constant. The plastic hysteresis energy relationship of Feltner and Morrow[9] is a special case of this more general theory when the temperature during testing is a constant. Since entropy gain accompanies every irreversible process and since fatigue damage is thought to result from plastic straining which is an irreversible process, the critical entropy gain hypothesis is intuitively pleasing as well. Fatigue is viewed as a random process whereby irreversible deformations accumulate until molecular bonds break down and cracks form. Entropy gain is assumed to be a measure of this accumulated damage. This paper describes an analysis of the formation of cracks, more precisely crack nucleation. This viewpoint addresses fatigue in two stages; crack nucleation whereby cracks suddenly appear in a previously unflawed material, and crack propogation whereby cracks grow until the part fractures. The entropy threshold is believed to hold in both cases, but the theory of fracture mechanics must be used to analyze the crack propogation.

Reference[32] explains a new stochastic mathematical model for internal friction. The irreversible part of the dissipated hysteresis energy is derived and used to generate strain amplitude versus cycles to failure curves for 6061-T6 Aluminum. Favorable comparisons between estimates of the model and experimental data are indicated. That analysis was conducted for constant temperature and was based on the

assumption that the yielding phenomenon does not change substantially with time. In this paper, the critical entropy threshold is derived on the basis of irreversible energy lost during static fracture and time and temperature dependence of the fatigue phenomenon is evaluated.

The phenomena of crack nucleation, material damping, and impurity concentration have been related to the diffusion mechanism in References[33]-[37]. Reference[34] uses partial differential equations for dislocation loop length distribution which are of the same form as the Fokker-Planck equation for the diffusion mechanism. This time-dependent probability distribution function results in an equation identical in form to Whiteman's[38] relationship for random yielding. It is therefore consistent to interpret fatigue damage as the mechanism of gradual diffusion of impurities within the material into local flaw concentrations observable as microscopic cracks. This diffusion of submicroscopic flaws (impurities) is a function of stress level, time, temperature, and initial impurity distribution. In this paper, the random process of fatigue damage is hypothesized to consist of the local concentration of flaws by diffusion leading eventually to the sudden appearance of randomly distributed populations of cracks.

The critical entropy threshold of fracture is defined in terms of the irreversible (plastic) part of the static fracture energy. This entropy threshold is assumed to be a material constant and is simply a measure of the amount of irreversibility (damage) a given material can withstand

before molecular bonds break. The entropy gain of fracture can be calculated using Whiteman's[38] equations for static stress. In a quasi-static tensile test, the temperature would not be expected to increase by very much so the local average temperature is assumed to be approximately constant during quasi-static fracture. The critical entropy threshold of fracture is therefore related to the irreversible part of the fracture energy by[18]:

$$S_f = \int_0^{\varepsilon_f} \sigma \varepsilon P(\varepsilon) d\varepsilon / \theta \quad . \tag{1}$$

This entropy threshold of static fracture is a random variable and the variability can be quantified by a confidence interval. The confidence interval for the entropy threshold just comes from the variance of the plastic stress, ν_p:

$$S_{fu} = \int_0^{\varepsilon_f} [\sigma \exp(2\nu_p)] \varepsilon P(\varepsilon) d\varepsilon / \theta \ , \tag{2}$$

$$S_{fl} = \int_0^{\varepsilon_f} [\sigma / \exp(2\nu_p)] \varepsilon P(\varepsilon) d\varepsilon / \theta \tag{3}$$

Combining equations (1)-(3) with equation(13) of Reference[32], the time required to exceed the critical entropy threshold can be estimated. This is demonstrated in Figure 1 where N_{fu}, N_f, and N_{fl} are the upper confidence interval, mean, and lower confidence interval of the time to crack nucleation, respectively. The result for constant

temperature is shown in Figure 6 of Reference[32], where the upper and lower confidence intervals are qualitative in nature and are based on the assumption that the variance of loss factor is proportional to the variance of dynamic plastic strain. The agreement of the fatigue crack nucleation lifetime estimate with data is encouraging. Although temperature increase may not strongly influence shorter fatigue lives, it is necessary to investigate the temperature rise during high-cycle fatigue. The temperature- and time-dependence of fatigue must therefore be addressed.

Equation(1) of Reference[32] is the assumed form of the random yielding probability distribution function as given by Whiteman[38], but also includes additional parameters to quantify the gradual changes in the diffusion mechanism with time and temperature. The parameter β comes from the Fokker-Planck equation and indicates that the mean yield strain can be expected to decrease with time. The variance of the random yielding during fatigue is also expected to change with time and temperature as described below:

$$\sigma_0{}^2=D_0\exp[K_\theta(\theta-\theta_m)/\theta]/\beta\{1-\exp[-2\beta(t+K_tN_f/f_i)]\}. \qquad (4)$$

In equation(4), K_t is a constant which makes the initial impurity concentration yielding a non-zero initial yielding

variance. The choice for the parameter K_t controls the time-dependent variance of the random yielding after long fatigue lifetime. Justification for the temperature dependence of the variance described in equation(4) is discussed in References[36], [40]-[42], along with guidance for choosing K_θ.

It is necessary to analyze the effect of long time and temperature rise on fatigue crack nucleation lifetime estimates. It is expected that Figure 6 of Reference[32] will be accurate for moderate fatigue lifetime and that time and temperature effects will only be apparent after high-cycle fatigue. It is also necessary to select values for the parameters K_t and K_θ. The thermodynamic analysis of base-excited cantilever beam specimens excited at resonance in vacuum will be presented in the next section, providing a data base for these parameter selections.

THERMODYNAMIC ANALYSIS OF FATIGUE SPECIMENS IN VACUUM

Figure 6 of Reference[32] is an encouraging preliminary result which indicates that assuming constant temperature during fatigue based on the critical entropy threshold yields crack nucleation estimates which are consistent with data, especially for moderate testing times. However, the diffusion mechanism reveals that the mean and variance are functions of time and temperature. It is therefore essential to investigate the influence of time and temperature on high-cycle fatigue life. In this section, a thermodynamic analysis of base excited cantilever beam specimens vibrating at resonance in vacuum is given and predicted temperature rise is compared to measured temperature increase. The data were collected according to procedures described in Reference[42].

Typical base-excited cantilever beam specimens always fracture at the base where the strain is a maximum. However, the tests were terminated after a two percent decrease in natural frequency which always occurred long before fracture, corresponding to a crack just being visible at the base. This practice was adopted because energy dissipation rate increased rapidly as a crack first appeared, and resonance became very difficult to maintain. Such a procedure is also consistent with this analysis for fatigue crack nucleation. Since the beam vibration is maintained precisely at resonance, the kinetic energy is equal and opposite to the elastic strain energy stored and all of the vibratory energy

is dissipated within the material. With a full vacuum maintained, the convection heat transfer term would be zero. For the low temperature rise typically observed in the laboratory, the radiation heat transfer would be negligible, and the only heat transfer present would be by conduction. Furthermore, all of the heat developed within the material must eventually be conducted back through the base, providing a means of analyzing temperature rise during these tests.

Figure 2 is a schematic depicting an infinitesemal element of the beam at some arbitrary position x. The energy dissipation rate at x is just:

$$dD=b\pi E\eta_s\varepsilon^2dy \tag{5}$$

For strains in the vicinity of yielding, loss factor is also a function of strain amplitude. It is therefore necessary to include the nonlinear dependence of loss factor on strain amplitude. For purposes of this thermodynamic analysis, mean loss factor was approximated as a constant plus a power of strain:

$$\eta_s=\eta_o + K\varepsilon^r.$$

Using Figure 4 of Reference[32], r and K are approximated to be 3.08 and 4.10×10^5, respectively. Then equation (5) becomes:

$$dD=b\pi E(\eta_o+K\varepsilon^r)\varepsilon^2dy$$

Strain is also a function of position within the cross section. Assuming that for moderate strain amplitudes plane sections remain approximately plane, the infinitesemal energy rate is just:

$$dD=b\pi E[\eta_o(2y\varepsilon_o/h)^2+K(2y\varepsilon_o/h)^{r+2}]dy$$

Now integrating to get the total energy dissipated at some position x,

$$D(x)=2\pi bE[\eta_o h\varepsilon_o^{\ 2}/6 + Kh\varepsilon_o^{\ r+2}/2(r+2)] \tag{6}$$

To calculate the total energy dissipated in the beam, equation (6) must be integrated along the length of the beam:

$$D_t=2\pi bE[\eta_o h/6 \int_o^{\ell} \varepsilon_o^{\ 2}dx + Kh/2(r+2) \int_o^{\ell} \varepsilon_o^{\ r+2}dx$$

Strain will also be a function of position but is proportional to the second derivative of the mode shape. Letting ε_s be the strain amplitude measured at the base of the cantilever beam, the total energy dissipation rate is:

$$D_t=2\pi bE[\eta_o h\varepsilon_s^{\ 2}/6 \int_o^{\ell} [\phi_i''(x)/\phi_i''(0)]^2 dx +$$

$$Kh\varepsilon_s^{\ r+2}/2(r+2) \int_o^{\ell} [\phi_i''(x)/\phi_i(0)]^{r+2}dx \tag{7}$$

Equation (7) represents the total energy which must be converted into heat or expended in damaging the material. A thorough inspection of equation (13) of Reference[32] reveals that for the moderate strain amplitudes of this analysis the plastic part of the dynamic strain amplitude is always less than a fraction of a percent of the total strain amplitude. This is consistent with the traditional interpretation that dissipated energy is converted into heat; for all practical purposes the small percentage of energy expended in damaging the material is negligible, according to this model. Therefore, equation (7) is considered to be the total heat source.

Now solving for the temperature rise in the form of heat stored in the material:

$$\partial\theta/\partial t = 2\pi E\{\eta_o h\varepsilon_s^{\,2}/6\int_0^{\ell}[\phi_i''(x)/\phi_i''(0)]^2 dx + K h\varepsilon_s^{\,r+2}/2(r+2)\int_0^{\ell}[\phi_i''(x)/\phi_i''(0)]^{r+2} dx\}/\rho C_p(hl+h_o l_o) \quad (8)$$

Figure 3 is a plot of temperature increase during fatigue tests in vacuum versus dimensionless time as compared to data. The agreement between predicted and calculated temperature increase is good. Figure 4 is a plot of strain amplitude versus cycles to crack nucleation including temperature rise during fatigue, and verifies the assumption that temperature rise during fatigue testing in vacuum has a small effect for short fatigue lives. For high-cycle fatigue

the temperature rise increases the slope of the entropy rate curve. The parameters used in this mathematical model for fatigue crack nucleation are summarized in Table I.

This provides experimental support for the hypothesis that the local entropy gain during fatigue is a constant at local failure. Although it is not possible with conventional laboratory techniques to accurately measure this entropy gain, experimental support has been provided for loss factor calculations, fatigue life estimates, and temperature rise during fatigue testing in vacuum. Additional experimental support for the truth of the entropy hypothesis is expected to develope as more research is completed.

At this point it is possible to address the temperature dependence of fatigue life estimates. Figure 5 is a plot of mean lifetime to fatigue crack nucleation for three different testing temperatures. The predictions are in substantial agreement with the trends outlined in Reference[43].

DISCUSSION OF RESULTS

A theoretical basis for the nucleation of fatigue cracks has been proposed, resulting in a new mathematical model of the fatigue mechanism. Experimental support for the validity of the entropy threshold of fracture has been presented. This mathematical model of fatigue is an important and significant contribution, not only because it reproduces experimental results for temperature rise, loss factor, and fatigue life estimates including confidence intervals, but because it has the capability of modeling temperature dependence of fatigue. In addition, this model has the capability of modeling the endurance limit. If the probability distribution function is truncated at some low strain level, then strains below that level will be elastic; the plastic part of such a low strain level would be zero, yielding an infinite fatigue life. In addition, an appropriate selection of the parameter K_t can influence the shape of the high-cycle fatigue curve. If K_t is large, the plastic part of the strain would not change substantially during high cycle fatigue. Likewise, the parameter K_θ influences the effect of temperature rise on the high-cycle part of the curve.

This mathematical model of fatigue is a general one describing the breakage of molecular bonds leading to strength degradation. As such, it has the potential for application to the phenomenon of crack propogation. In this study, crack nucleation is viewed as a random process whereby microscopic cracks suddenly appear in the material whenever

the local fracture entropy threshold has been exceeded. This sudden appearance of a crack would be described mathematically by a delta function. However, the propogation of the crack would occur from the same random yielding phenomenon; as the local fracture entropy threshold is exceeded at the leading edge of a crack, the crack would extend due to the local breakage of molecular bonds within the material. The analysis of crack propogation by the entropy threshold is being initiated at this writing.

Figure 6 is a plot of lifetime to crack nucleation versus strain amplitude for steel as compared to the empirical curves of References[9], [11], and[44]. Parameters for steel are listed in Table II. The critical entropy threshold is seen to compare consistently with prior published work, especially with Reference[11] which is based on a plastic hysteresis energy relationship. When temperature increases during fatigue as would normally be the case, the critical entropy threshold would alter the shape of the high-cycle fatigue curve, perhaps explaining why Martin[10] did not find Feltner and Morrow's results universally applicable.

The existence of a mathematical model accurately describing the physical phenomenon of fatigue is a significant advance. Once the accuracy of this model is established, the possibility of computer-aided structural design for maximum longevity exists. Such a design procedure might provide more efficient engineering design and developement, testing, and evaluation for new structures, as

well as more accurate replacement criteria for maintenance. It is likely that an accurate mathematical model for fatigue including the effects of temperature, environment, and loading history could provide significant cost savings in structural design, testing, and maintenance.

The choice of parameters of the random yielding model described in Reference[32] provides excellent agreement to measured data. However, the gradient search technique requires a lot of computer time and does not yield a direct measurement of the material fracture entropy. Reference[45] shows acoustic emission rate as a function of strain yielding a distribution function strikingly similar to the random yielding probability distribution function of the current mathematical model for fatigue. Since acoustic emission is believed to result from energy release during plastic deformation, it is likely that acoustic emission can be used to measure directly the critical entropy threshold of fracture during tensile tests. The significance of this would be that material fatigue could then be quantified by this single number which would be a material property.

It is recommended that additional analysis be conducted as a further test of the hypothesis that the critical entropy threshold is a necessary and sufficient condition for local material failure. Although these results for the formation of cracks are encouraging, a comprehensive analysis of structural fracture requires a model for crack growth. The critical entropy threshold is believed to be a necessary and sufficient condition for crack growth as well, and experimental and analytical evidence for this is needed. The

temperature dependence of damping, fatigue and fracture is an important engineering application and the model's capability of describing temperature dependence needs to be explored. Even more significantly, this theoretical basis for local failure has the capability for allowing analysis of arbitrary loading conditions and this aspect should be tested further. Finally, since the critical entropy threshold of local failure is a general relationship, other structural materials should be studied.

SUMMARY

A mathematical model for the fatigue mechanism in metals has been described. Physically, fatigue is explained in terms of two phases: first, the gradual diffusion of submicroscopic impurities into local concentrations of flaws is manifested as the sudden appearance of crack populations within the material. Second, the extension of cracks during dynamic loading is traditionally analyzed using the methods of fracture mechanics and is known as crack propogation. Both of these phases are considered to occur because of the exceeding of a critical entropy threshold which is a constant at local material failure. This mathematical model for fatigue is significant since it is possible to model time and temperature dependence of fatigue under arbitrary loading. Although the present analysis was conducted for uncracked metal specimens and therefore corresponds to crack nucleation, the approach is a general one and can be extended to the case of crack propogation using the methods of fracture mechanics.

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TABLE I. SUMMARY OF PARAMETERS USED IN THE MATHEMATICAL MODEL OF FATIGUE CRACK NUCLEATION IN 6061 T6 ALUMINUM

Parameter	Definition	Source	Numerical Value
C_p	Thermal Capacitance	Handbook	905.04 J/kg K
E	Young's Modulus	Handbook	69×10^9 Nt/m^3
D_o	Diffusion Constant	References[36]&[39]	1.919×10^{-9}/sec
γ	Low-strain exponential factor constant	Reference[32]	-0.56
β	Parameter describing time-dependent decrease of strength during high-cycle fatigue	References[17]&[33]	8.40×10^{-13}/sec
θ_m	Material melting temperature	Handbook	855.2 K
ρ	Density	Handbook	2712.5 kg/m^3
K_θ	Constant showing temperature dependence of diffusion	-----	0.2
K_t	Constant showing time dependence of diffusion	-----	10.0
σ_o	Initial standard deviation of random yielding	Reference[32]	0.67514
ε_m	Mean yield strain	Reference[32]	35.784×10^{-3}
ε_f	Maximum strain level before cracks form in a static tensile test	Reference[32]	0.02

TABLE II. SUMMARY OF PARAMETERS USED IN THE MATHEMATICAL MODEL OF FATIGUE CRACK NUCLEATION FOR STEEL

Parameter	Definition	Source	Numerical Value
C_p	Thermal Capacitance	Handbook	472.2 J/kg K
E	Young's Modulus	Handbook	196.65×10^9
D_o	Diffusion Constant	References[36]&[39]	3.305×10^{-9}/sec
γ	Low-strain loss factor constant	Reference[32]	-0.56
β	Parameter describing time-dependent decrease of strength during high-cycle fatigue	References[17]&[33]	1.45×10^{-12}/sec
θ_m	Material melting temperature	Handbook	1783.0 K
ρ	Density	Handbook	7743.6 kg/m^3
K_θ	Constant showing temperature dependence of diffusion	-----	0.2
K_t	Constant showing time dependence of diffusion	-----	10.0
σ_o	Initial variance of random yielding	Reference[32]	0.67514
ϵ_m	Mean yield strain	Reference[32]	39.926×10^{-3}
ϵ_f	Maximum Strain level before cracks form in a static tensile test	Reference[32]	0.03

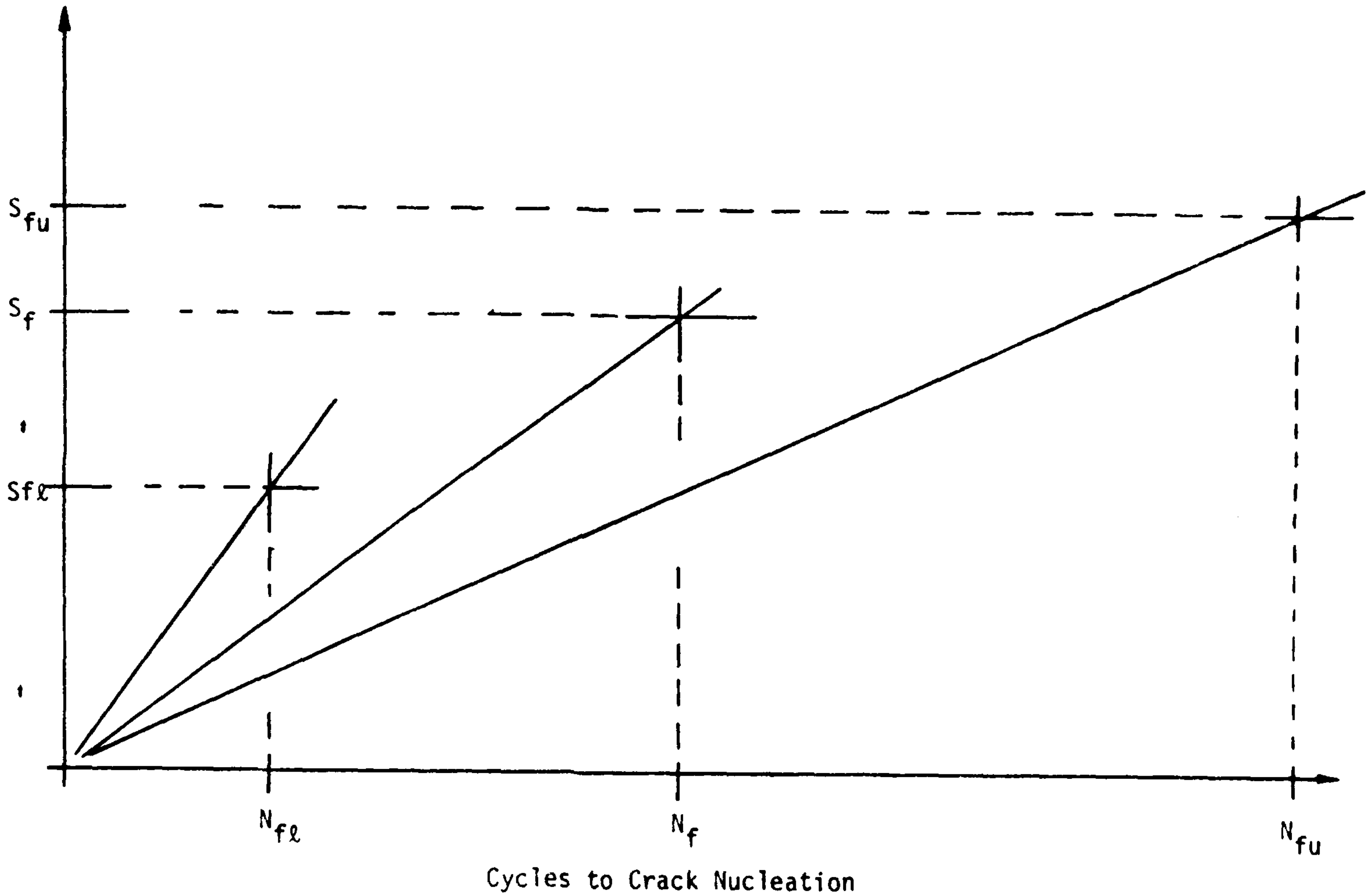

Figure 1-1. Integration of Entropy Rate Yielding Fatigue Crack Nucleation Lifetime Confidence Intervals Including Variation of the Entropy Threshold.

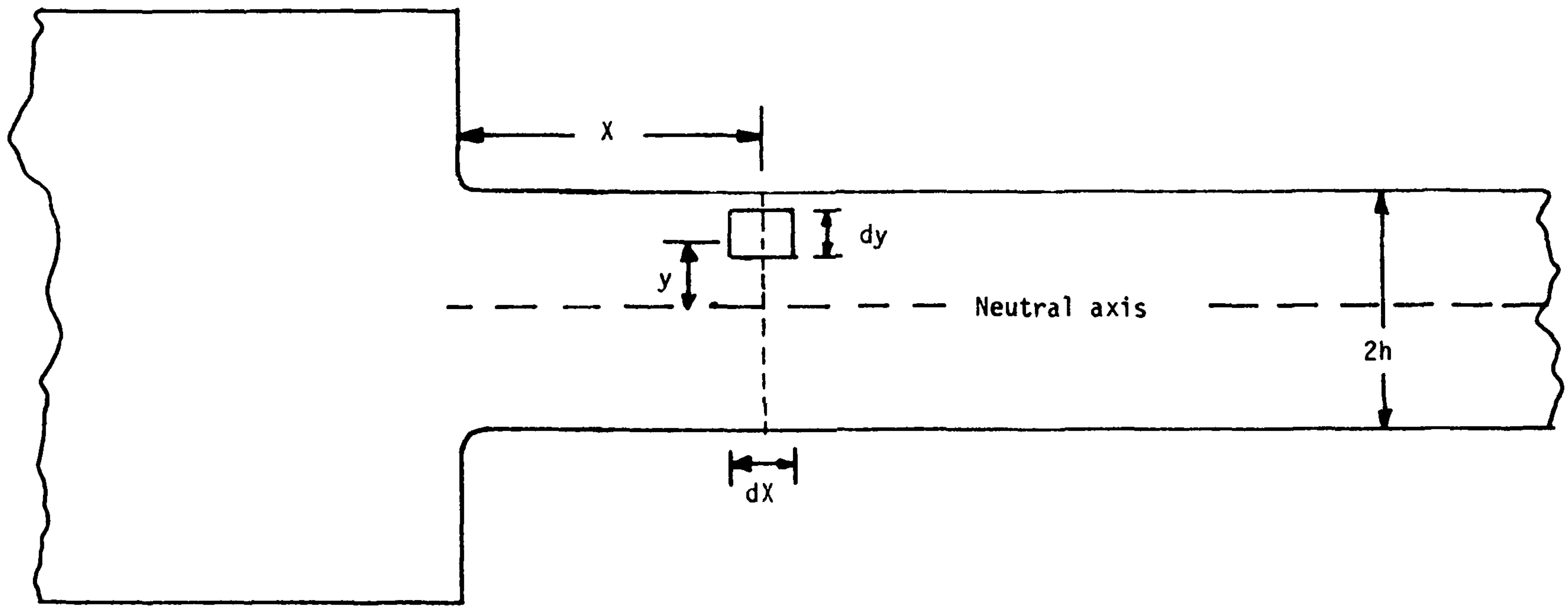

Figure 1-2. Schematic Drawing Indicating Energy Dissipation Rate in an Infinitesemal Beam Element.

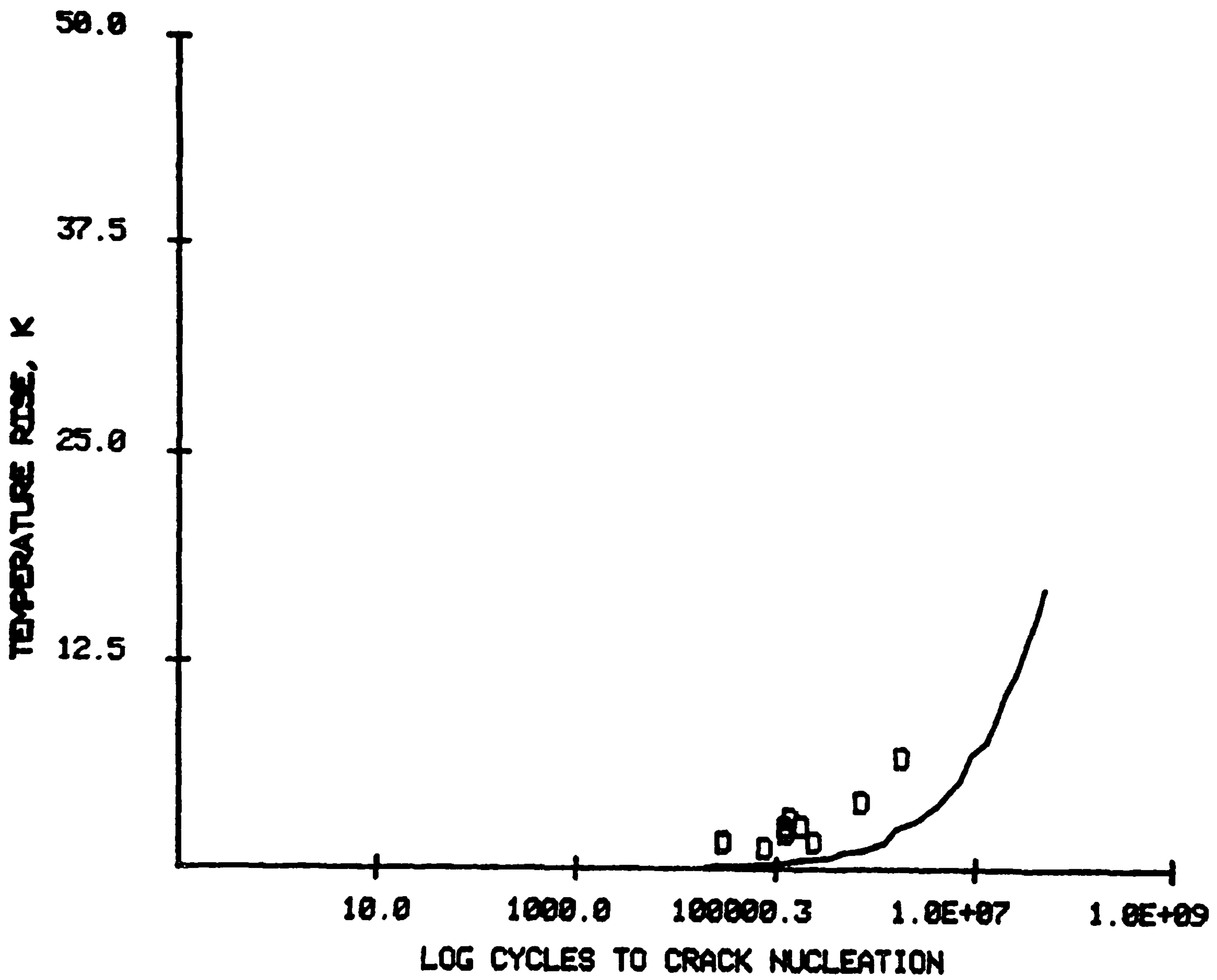

Figure 1-3. Temperature Increase Versus Dimensionless Time (Cycles) as Compared with Data for 6061-T6 Aluminum According to Reference [36].

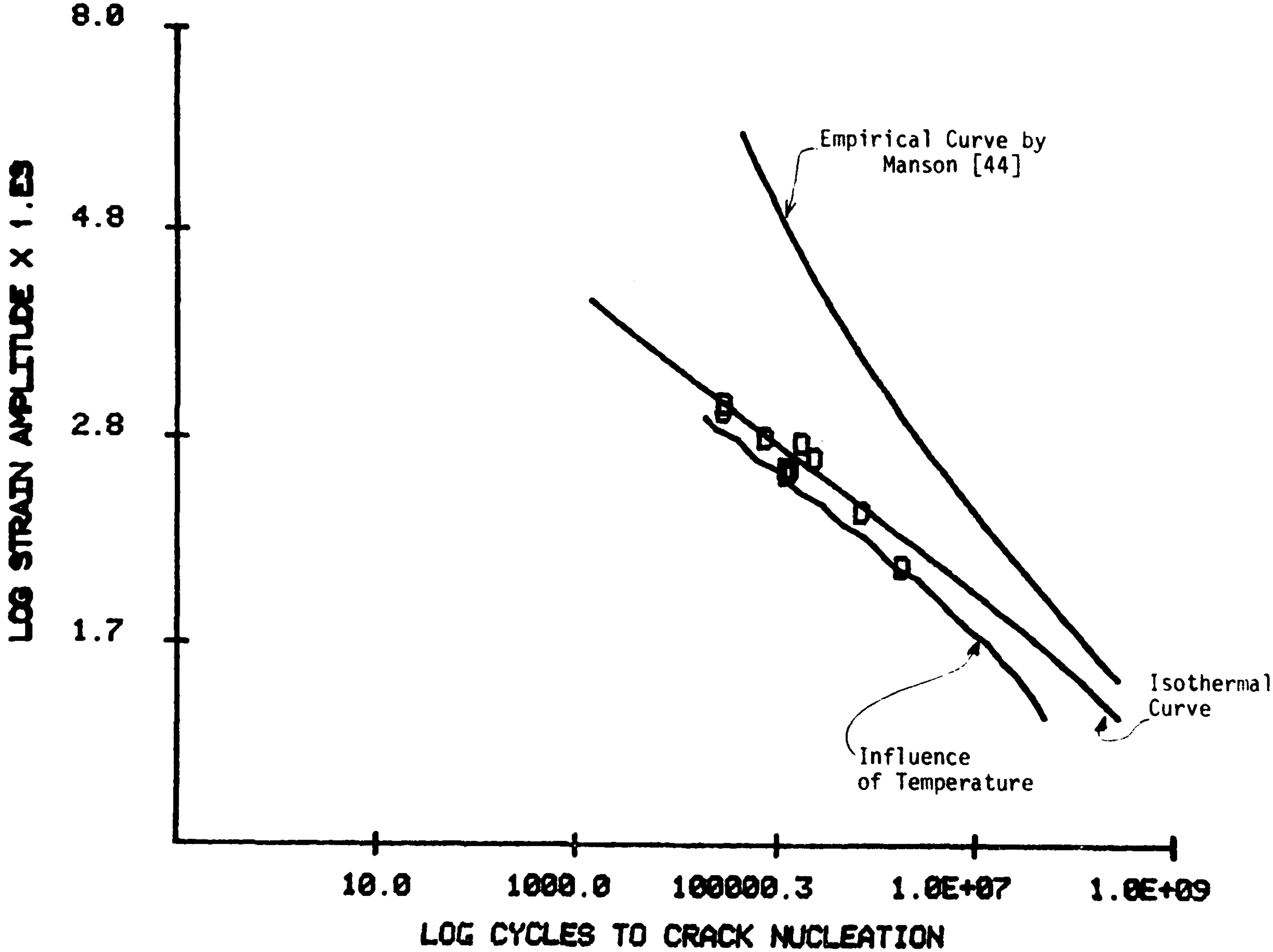

Figure 1-4. Strain Amplitude Versus Cycles to Fatigue Crack Nucleation for 6061-T6 Aluminum Including the Influence of Temperature Rise on High-Cycle Fatigue.

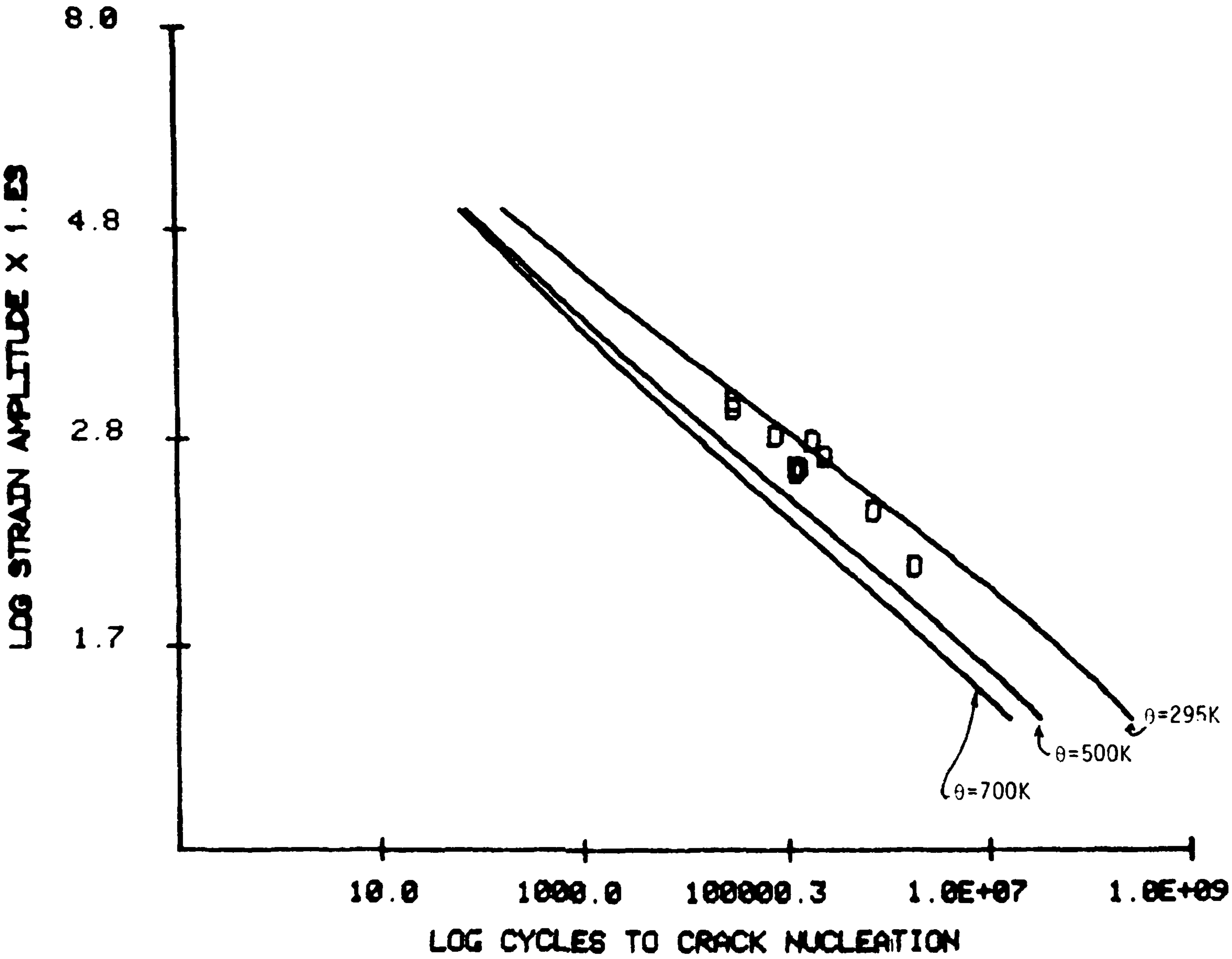

Figure 1-5. Temperature Effects on Fatigue of 6061-T6 Aluminum as Predicted by the Current Mathematical Model for Fatigue.

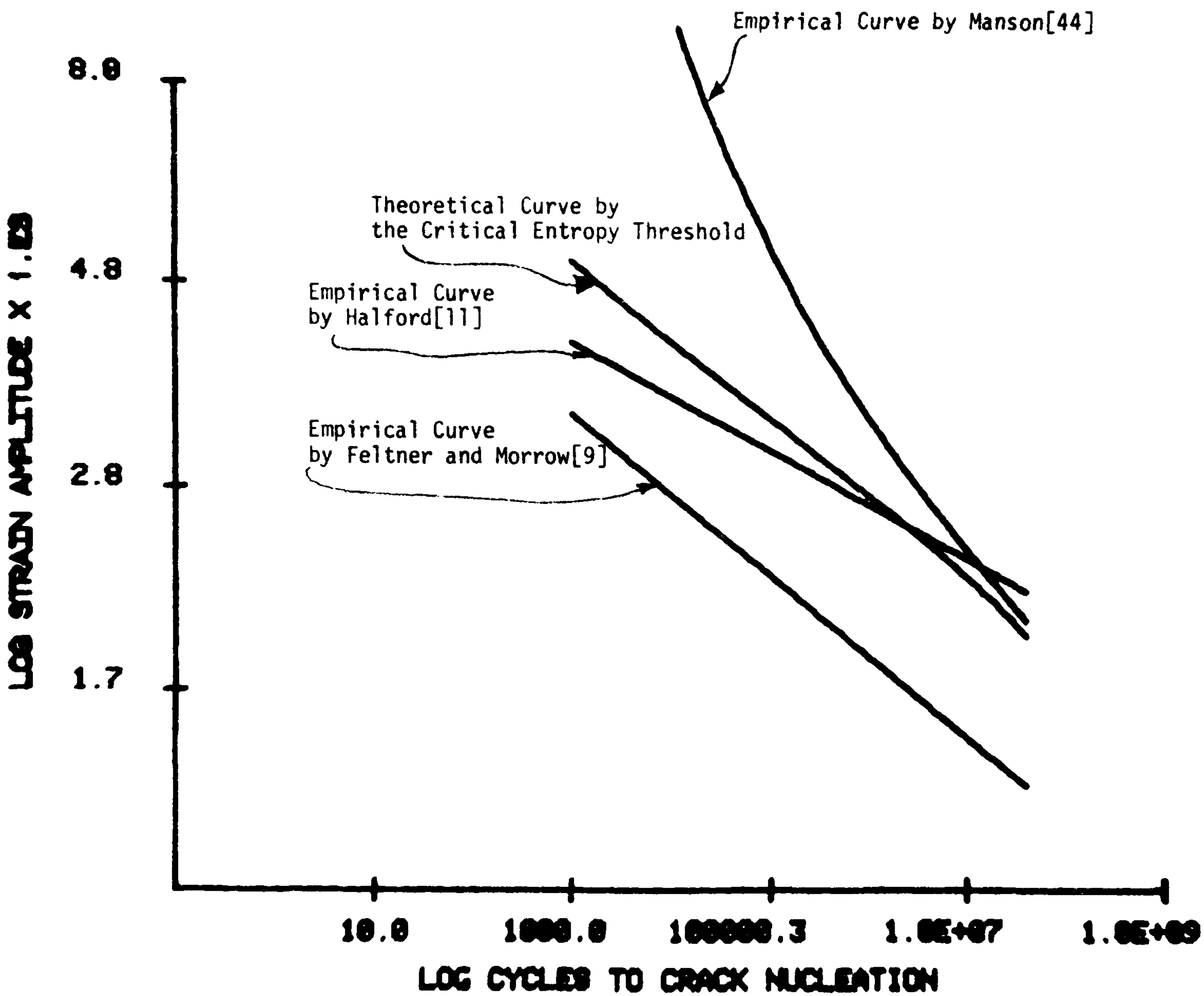

Figure 1-6. Fatigue Crack Nucleation Curves for Steel Compared With the Results Given in References [9], [11], and [44].

A STOCHASTIC MODEL FOR MATERIAL DAMPING BASED ON RANDOM YIELDING

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ABSTRACT

A mathematical model for internal friction in metals based on random yielding is described. It is demonstrated by comparison to laboratory data that the model accurately estimates strain-amplitude and frequency dependence of loss factor. In addition, since the model is based on a random phenomenon, variability can be described. A qualitative confidence interval is defined which agrees favorably with experimental data. The model also accurately predicts the frequency dependence of loss factor at low strain amplitudes. The significance of this new damping model lies primarily in its application with a new theory for the fundamental mechanism of material fatigue. Based on the hypothesis that fatigue is modeled by exceeding the critical fracture entropy threshold, the familiar strain-amplitude versus cycles to failure relationship results. Although this analysis was based on an assumption that the material have no cracks present, the approach is a general one and the model for loss factor can easily be extended to the case of cracking in the material. Finally, although this analysis was conducted for metals, the general phenomenon of random yielding is thought to apply to other materials as well.

INTRODUCTION

The dissipation of structural vibratory energy is due to several different mechanisms among which are fluid viscosity by interaction with fluid surroundings, friction in joints, and internal friction due to microscopic rubbing. The material damping from internal friction is usually called the hysteresis damping and has been the subject of research for many years. This hysteresis damping is normally orders of magnitude smaller than other active damping mechanisms so for the purpose of determining resonant amplitudes is usually neglected. There are at least two exceptions to that. First, in space applications, the absence of air renders the hysteresis damping predominant in some cases. Second, for large deflections the hysteresis damping increases significantly since it is usually an increasing nonlinear function of strain amplitude.

The area of vibration damping is an important one for many reasons. In the calculation of forced vibration amplitude, the resonant amplitude is normally orders of magnitude greater than the static deflection. For this reason, the resonant amplitude is usually used to estimate the maximum dynamic stress amplitude. Unfortunately, the uncertainty in loss factor significantly affects the uncertainty in resonant amplitudes. In order to develope accurate models for forced vibration response, it is essential to understand the mechanisms of structural damping.

The practical implications of dynamic response calculations are two-fold: first, as resonant amplitudes increase, sensitive electronic components commonly installed in modern structures are less likely to function properly, and in some cases come apart. Second, under sustained high amplitudes of forced vibration the structural members themselves are highly susceptible to dynamic fracture.

Reference[1] contains an excellent discussion of material damping, identifying two different major mechanisms of internal friction. First, there is a rate-insensitive hysteresis in the elastic-plastic response of the material. Second, there is a rate-sensitive heat flow in the material. For torsion, only shear deformations would be present, and Dawson[1] showed that a constitutive law of elasticity leads to several damping coefficients. Those damping coefficients come from a Taylor series expansion of the constitutive law and represent dependence on stress level. The first coefficient would be the familiar loss factor, which is constant with respect to stress level. The second coefficient would represent dependence on stress squared, and so on. This approach is significant since no shape of the hysteresis loop is assumed, and a description of plasticity below the conventional yield point of the material is given.

References[2]-[4] are applications of similar kinds of equations to describe the internal hysteresis damping of materials. All of these include a dynamic stress-strain relationship in terms of loss and storage modulii which gives rise to a phase shift between stress and strain. In view of

the many different approaches to material damping analysis, it is necessary to define common terminology. The loss factor is normally defined as the ratio of energy dissipated to energy stored per cycle, and is a material constant relating stress to strain, independent of boundary conditions or damping mechanism. In this paper loss factor is used as a measure of internal friction.

The determination of resonant stress amplitudes has been a major motivation for analysis of damping since dynamic stress amplitudes can be orders of magnitude greater than static amplitudes in many cases. This emphasis on dynamic fracture is expected to continue, but lower dynamic amplitudes of loading are also directly related to fatigue fracture over the long term. This relationship between dynamic stress and fracture involves more than just the stress amplitude, however; a thermodynamic analysis of fatigue involves the energy dissipation rate directly. In fact, a number of papers in the open literature have related material hysteresis damping directly to fatigue.

References[5]-[9] relate the theory of dislocation mechanics to material hysteresis damping. This view attributes hysteresis to the microscopic diffusion of impurities within the material. Since the concentration of impurities and their motion are random, this approach identifies internal friction as an inherently random phenomenon. Reference[10] presents the theory of a new hypothesis for the fundamental mechanism of fatigue based on this diffusion mechanism, using a model for the hysteresis

loop by Whiteman[11] to derive diffusion equations for random yielding. Whiteman[11] argued that his equation for the hysteresis loop is intimately related to the fatigue process, but failed to derive the precise relationship. Reference[10] provides the relationship that Whiteman[11] was searching for.

Others have investigated the relationship between internal friction and fatigue[12]-[16]. Reference[12] analyzed internal friction as related to dislocation motion. References[13] and[14] proposed that the plastic part of the dissipated hysteresis energy is a criterion for fatigue, but both failed to provide a reliable theoretical relationship. References[15] and[16] are similar applications to non-metals and use the relationship between plastic strain rate and permanent damage as a criterion for fatigue. Reference[10] extends that approach to an irreversible thermodynamic analysis of the fatigue process and proposes that it is not dissipated plastic hysteresis energy which influences fatigue, but the entropy gained during fracture which is constant for a given material. This new hypothesis is proposed as a mathematical model for the fatigue mechanism, and is consistent with the results of previous researchers. The details of that derivation are found in Reference[10], but that analysis uses a mathematical model for internal friction capable of calculating the reversible and irreversible contributions to energy dissipation rate. This new mathematical model of internal friction based on random yielding is capable of reproducing the irreversible

contribution of dissipated hysteresis energy and is the topic of the present paper.

Whiteman's[11] equation for the hysteresis loop is modified in the present analysis to include damping below the yield point and to define internal friction in terms of loss factor. The fact that this approach views internal friction as a random process is significant since expected variations in loss factor measurements can be modeled directly. In addition, the perception of internal friction damping may mature through this approach, as the fundamental differential equations of motion would include coefficients randomly changing with time. Reference[17] concluded that when variations about the mean are small, the differential equation can be approximated in terms of the mean and variance of the randomly varying coefficient. This assumption appears to be consistent for the present analysis of structural damping.

In the next section the stochastic model for material damping is derived and related to known frequency- and strain amplitude- dependence of loss factor. It is assumed that there are no engineering size cracks present in the material. In this manner, a material damping model is derived which predicts frequency, amplitude, time and temperature dependence of hysteresis energy dissipation rate.

INTERNAL FRICTION BASED ON RANDOM YIELDING

According to Whiteman[11], the probability distribution function for random yielding is approximately log-normal for Aluminum. From this equation, he calculated a static stress-strain relationship and hysteresis loop which include explicitly the plastic part of strain amplitude. His equation is modified in this paper to include phase lag between stress and strain at low strain-amplitudes by adding a term as given below:

$$P(\epsilon)=A(\omega)\exp[-a|\dot{\epsilon}|]+ \exp\{-[\log\epsilon-\log(\epsilon_m)\exp(-\beta t)]^2/\sigma_o^2\}/\sqrt{2\pi}\sigma_o \qquad (1)$$

The second term in equation(1) is identical to Whiteman'2[11] equation except that the diffusion mechanics has been modeled providing that the mean and variance change with time as described in detail in Reference[10]. The first term in equation(1) is new and includes a frequency dependent elastic hysteresis which is due to the phase lag between stress and strain. For large a, the first term approaches a delta function and the corresponding mean plastic strain is zero.

When the loading is a function of time, the phase between the stress and strain can be represented on the complex plane as shown in Figure 1. The parameters a and A can be selected to model frequency dependence of loss factor:

$$\eta_e=(A/\omega a)=\eta_o(\omega/\omega_i)^{\gamma} \qquad (2)$$

Referring to Figure 1, the storage modulus due to plastic strain can be calculated by substituting $\varepsilon=\varepsilon_s$ into Whiteman'2[11] equation for the hysteresis loop:

$$\sigma_s=[1-\int_{-\varepsilon_s}^{\varepsilon_s} P((\eta+\varepsilon_s)/2)d\eta/2+\int_{-\varepsilon_s}^{\varepsilon_s} \eta P((\eta+\varepsilon_s)/2)d\eta/2\varepsilon_s]E\varepsilon_s \qquad (3)$$

Equation(3) represents the storage modulus for high strain amplitudes and reflects the fact that the dynamic probability structure of yielded elements is displaced[11]. Combining equations(1) and(3), the dimensionless storage modulus is:

$$E_s = \sigma_s/E\varepsilon_s \qquad (4)$$

The last term in equation(3) represents the irreversible, or plastic contribution of the storage modulus in the form of the expected value of the plastic strain. From Figure 1, the total loss factor is:

$$\eta_s(\varepsilon) = \eta_e + \eta_p(\varepsilon) \qquad (5)$$

When the strain amplitude is low, the dimensionless storage modulus in equation(4) will be approximately one, and the loss factor is entirely due to the low-amplitude phase shift described by the first term of equation(1). As strain amplitude approaches the yield point of the material, the dimensionless storage modulus will be less than one and an equation for the plastic contribution to the loss factor can

be calculated:

$$\eta_p^2 + 2\eta_e\eta_p - (1-E_s^2) = 0 \tag{6}$$

Equation(6) is in quadratic form and the positive root is:

$$\eta_p = -\eta_e + \sqrt{1 + \eta_e^2 - E_s^2} \tag{7}$$

Combining equations(5) and(7), the result is:

$$\eta_s(\varepsilon) = \sqrt{1 + \eta_e^2 - E_s^2} \tag{8}$$

Equation(8) is a closed form expression which models frequency-, strain amplitude-, time- and temperature-dependence of internal friction. Reference[10] contains a discussion of the time and temperature dependence of loss factor. This paper will concentrate on the frequency- and strain amplitude-dependence of loss factor. For low strain levels, E_s goes to one and η_s goes to η_e according to equation(8). As strain amplitude approaches the material yield strain, E_s becomes smaller than one and η_s increases. Figure 2 is a plot of loss factor versus sinusoidal strain amplitude for several frequencies using equation(8). Figure 2 represents a dimensionless description of frequency dependence of loss factor and reproduces the common interpretation that frequency dependence is only observable at low strains. The reason for that, according to this model, is that the loss factor due to random yielding

dominates at strain amplitudes in the vicinity of the yield point and higher. Figure 3 is a plot of low-strain loss factor versus frequency including a least squares curve fit to data and is the source of the frequency exponent of equation(2).

Equation(8) represents the mean value of the loss factor following the derivation of Whiteman[11]. This includes the implicit interpretation that loss factor is a random variable since plastic strain is modeled as a random variable. It is therefore possible to calculate the variance of loss factor to quantify the expected variability. Since the variation of loss factor is caused by the plastic strain, it is logical to assume that the variance of loss factor is proportional to the variance of dynamic plastic strain. The variance of the dynamic plastic strain is just the difference between the mean square and the square of the mean:

$$\nu_s^{\,2} = \int_{-\varepsilon_s}^{\varepsilon_s} \eta^2 P((\eta+\varepsilon_s)/2)\,d\eta/2 - \left[\int_{-\varepsilon_s}^{\varepsilon_s} P((\eta+\varepsilon_s)/2)\,d\eta/2\right]^2$$

Then the upper and lower confidence intervals for loss factor are assumed to be:

$$\eta_{su} = \eta_e + \nu_s \exp(C_\varepsilon \nu_s/\varepsilon_p) \qquad (9a)$$

$$\eta_{sl} = \eta_e + \nu_s/\exp(C_\varepsilon \nu_s/\varepsilon_p) \qquad (9b)$$

Equations(9) are based on the assumption that the plastic contribution to the loss factor is a log-normal random variable whose variance is approximately equal to the

variance of the dynamic plastic strain. Figure 4 is a plot of the strain amplitude dependent loss factor including confidence intervals using equations(8) and(9), along with data collected according to the procedures outlined in References[18] and[19]. Figure 4 is in substantial agreement with the trends described in Reference[12]. Note that according to this model uncertainty in loss factor increases as strain amplitude increases, consistent with the model for random yielding.

The nonlinear loss factor of Figure 4 is represented as a linearized dashpot system in Figure 5. The total loss factor of Figure 4 is represented as an equivalent dissipation element. Since a material element would have a common stress, the plastic and elastic dashpots are connected in parallel. The energy dissipated per cycle in the plastic part of the strain is therefore:

$$D_i = \int_0^{2\pi/\omega_i} \sigma(d\varepsilon_p/dt)dt = \pi E\eta_1\varepsilon_p^2$$

Substituting for η_1, the irreversible part of the dynamic strain energy is related to the entropy by the first law of thermodynamics[20]:

$$\theta\partial s/\partial t = \pi\ E\eta_s\varepsilon_s^2(\varepsilon_p/\varepsilon_s) \qquad (10)$$

Equation(10) contains the entropy rate during sinusoidal strain loading. If the temperature is approximately constant while testing in vacuum, equation(10) can be integrated in

closed form. By calculating the time required to exceed the critical entropy threshold of fracture described in Reference[10], the familiar strain-amplitude versus cycles to failure diagram in Figure 6 results. Figure 6 also indicates favorable agreement to the empirical equation published by Manson[21] and data collected using the procedure described in Reference[19]. The significance of this is that structural damage by fatigue is now quantifiable by a single material parameter, the critical entropy threshold of fracture.

An internal friction mathematical model based on random yielding has been derived which demonstrates both frequency- and strain amplitude- dependence of loss factor. Uncertainty in the loss factor is quantified by the variance of the dynamic plastic strain. Favorable agreement between the mathematical model and experimental data is demonstrated. In the next section, a description of the procedure used in selecting the parameters of the model is given.

PARAMETER SELECTION

The probability distribution function given in equation(1) includes two parameters which must be selected in order to complete the mathematical model of loss factor: the random yielding variance σ_0^2, and the mean yield strain ϵ_m. Additional parameters involving time- and temperature-dependence contained in the diffusion mechanics are described in Reference[10]. The procedure used in selecting ϵ_m and σ_o is described next.

The criteria to be used in selecting ϵ_m and σ_o are that the model predictions for loss factor and maximum stress agree with experimental data. That procedure requires some engineering judgement since the precise upper limit on the strain is not known. This mathematical model for internal friction is based on the assumption that there be no cracks present within the material. Therefore, the upper strain limit is to be selected such that the material yields without cracking. This upper strain limit will normally vary for different materials and is to be selected on the basis of engineering judgement.

It is generally believed that the amplitude dependence of damping begins to be important when the dynamic stress amplitude approaches the two percent yield point. However, there is no agreement as to the strain level where that amplitude dependence begins. Dawson[1] describes the amplitude dependence as beginning to be important when the stress amplitude is a sizeable percent of the yield stress of the material. Feltner and Morrow[13] mention a transition

region between the endurance limit and high stresses without specifying the point where amplitude dependence of loss factor begins. Mason'2[12] data shows an abrupt increase in loss factor somewhere below the yield point of the material.

Equation(7) represents the amplitude and frequency dependent loss factor predicted by the model, including the plastic part. The maximum static stress is:

$$\sigma_u{}^* = E\varepsilon_f[1-\int_o^{\varepsilon_f} P(\eta)d\eta] + E\int_o^{\varepsilon_f} \eta P(\eta)d\eta \qquad (11)$$

Since some stress relief would normally be expected just after a crack appears, the local maximum stress would normally be greater than the ultimate strength available from engineering handbooks. The ratio of the maximum stress to the ultimate strength is to be evaluated in selecting the model parameters by engineering judgement.

Equation(10) is the relationship between the constant entropy of fracture derived in Reference[10] and the lifetime at the endurance limit. The parameters of the material damping model are to be selected such that this lifetime agrees with the handbook value of N_f=5 x 10^8 cycles. In addition, data for loss factor are available for guidance in causing the model to correctly predict observed phenomenon.

In order to select σ_o and ε_m, gradient search techniques were used by defining the following function to be minimized:

$$J = [\pi E\eta_s(\varepsilon_e)\varepsilon_e{}^2(\varepsilon_p/\varepsilon_e)N_f/\theta_r S_f-1]^2 + [\eta_s{}^*/\eta_m - 1]^2 \qquad (12)$$

which occurs whenever $N_f = 5 \times 10^8$ cycles and $\eta_s{}^* = \eta_m$, where η_m is the measured loss factor near the transition to plastic strain. Using equation(8) the transition loss factor is arbitrarily selected as:

$$\eta_s{}^* = \eta_s(0.5\varepsilon_y)$$

The results of this procedure are summarized in Table I, and the last row represents the best agreement to laboratory data. Table I shows that as the maximum strain increases, $\sigma_u{}^*$ increases, as do ε_m and σ_o. Increasing maximum strain increases the slope of the $\varepsilon_s - N_f$ diagram of Figure 6. The maximum strain which yielded the best agreement to laboratory data without unreasonable local maximum static stress was selected.

In this section, the procedure for selecting the parameters of the mathematical model for internal friction based on random yielding is described and the resulting best parameters are given. In the next section, a discussion of the significant results provided by this new material damping model is presented along with recommendations for future research.

DISCUSSION OF RESULTS

Figures 3 and 4 represent estimates of loss factor using the frequency- and strain amplitude-dependent material damping model. The purpose of this model is not normally the calculation of resonant amplitudes as with many applications. Figure 4 clearly shows that as strain amplitude increases, damping increases. Although such an increase in damping would decrease the resonant amplitude, it would also be expected to result in fatigue damage. Rather, the purpose of this damping model is to estimate the irreversible part of the hysteresis energy dissipated per cycle in an attempt to quantify fatigue by the entropy threshold of fracture as described in Reference[10].

These results are believed to be general ones which can be readily extended to other practical cases. Although this analysis was for uncracked specimens, the phenomenon of random yielding is considered to be a universal local condition. After a crack has appeared, random yielding would still occur but would be significantly affected by the local stress concentration. Damping and fatigue could be analyzed using the methods of fracture mechanics. In addition, other metals and non metals could be investigated from the same random yielding viewpoint.

This material damping and fatigue model contains implicitly the capability for investigating the random dynamic response of structures. The time required for nucleation of cracks under random loading can be described, and using the methods of fracture mechanics the growth of the

crack under random vibration can also be described. A theoretical basis for the irreversible loss of energy under arbitrary loading can therefore be developed.

Table I provides some insight into the effect of the parameters variations. First of all, changing ε_f changes the slope of the strain-lifetime curve of Figure 6. It is therefore a straight-forward procedure to determine the set of ε_m, σ_o which satisfies equation (12) for a given ε_f. Then engineering judgement should be used to select the best fit to laboratory data. However, that procedure is complicated by the fact that as the fit to loss factor and fatigue life data improves, the maximum local static stress increases. According to Table I, this stress is always greater than the ultimate strength. It is therefore necessary to examine more closely the phenomenon under study.

If commonly used engineering data for ultimate strength comes from strain measuring instruments, then some average stress is actually indicated. Therefore it is logical that some local increase of stress would not be obvious in the data. Furthermore, this mathematical model for dynamic local failure is a description of the sudden appearance of a crack. The stress just before a crack appears would be greater than the stress just after the crack appears due to stress relief. The local increase in stress just before a crack appears is indicated in Table I and the most appropriate parameters are selected based on engineering judgement.

One final comment concerning this mathematical model of internal friction is in order. The low amplitude frequency

exponent, γ, in equation(2) resembles the fractional calculus approach of Reference[22]. This impacts the form of the differential equation where the frequency exponent corresponds to the order of a derivative. This model for internal friction might therefore prove useful in finite element modeling of complex structures for the purpose of providing a design estimate of the structural fatigue life.

SUMMARY

A new mathematical model for internal friction based on random yielding has been presented. The model accurately predicts both frequency- and strain amplitude- dependence of loss factor. Since the approach is based on a random phenomenon, the uncertainty in loss factor can be quantified, leading to a mathematical model for fatigue, including a quantification of uncertainty in fatigue as described in Reference[10]. Although this analysis was carried out for a material with no cracks present, the theory is a general one and can be extended to the case where cracks are present using the methods of fracture mechanics. Finally, the phenomenon of random yielding is believed to be a universal one and would be expected to apply to all structural materials.

ACKNOWLEDGEMENTS

This work was supported by the Office of Naval Research, ONR contract number N00014-82-K-0804. Appreciation is due Mr. K. N. Lin for conducting the gradient search procedure in selecting parameters, and to Mr. P. S. Chen for collecting the laboratory data.

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TABLE I. DYNAMIC RANDOM YIELDING MODEL PARAMETERS USING DIFFERENT UPPER STRAIN LIMITS FOR 6061-T6 ALUMINUM

ε_f	σ_u^*/σ_u	$\varepsilon_m \times 10^3$	σ_o
0.015	2.614	22.135	0.56233
0.020	3.421	35.784	0.67514

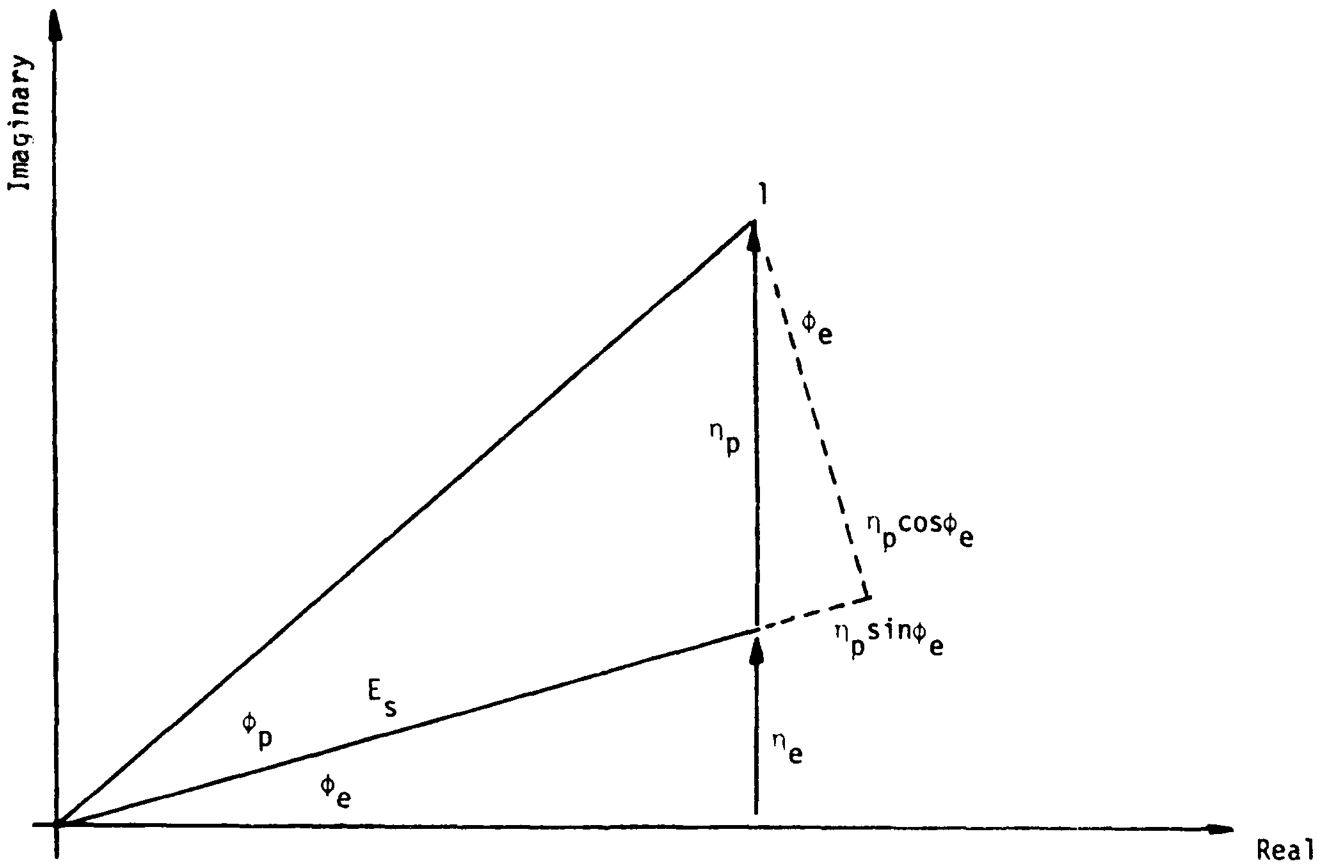

Figure 2-1. Relationships Between Plastic and Elastic Contributions to Stress and Strain on the Complex Plane.

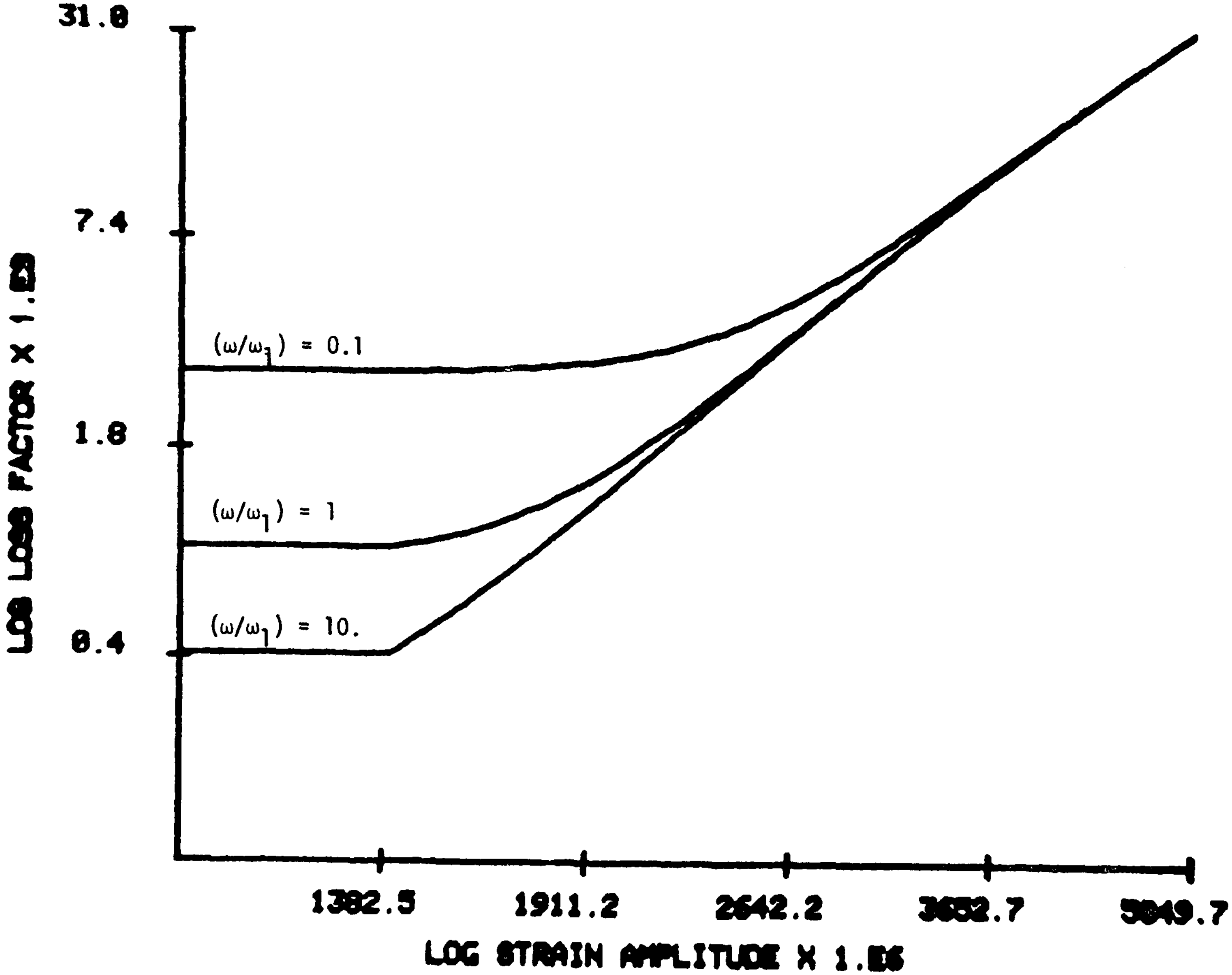

Figure 2-2. Prediction of Frequency Dependence of Loss Factor.

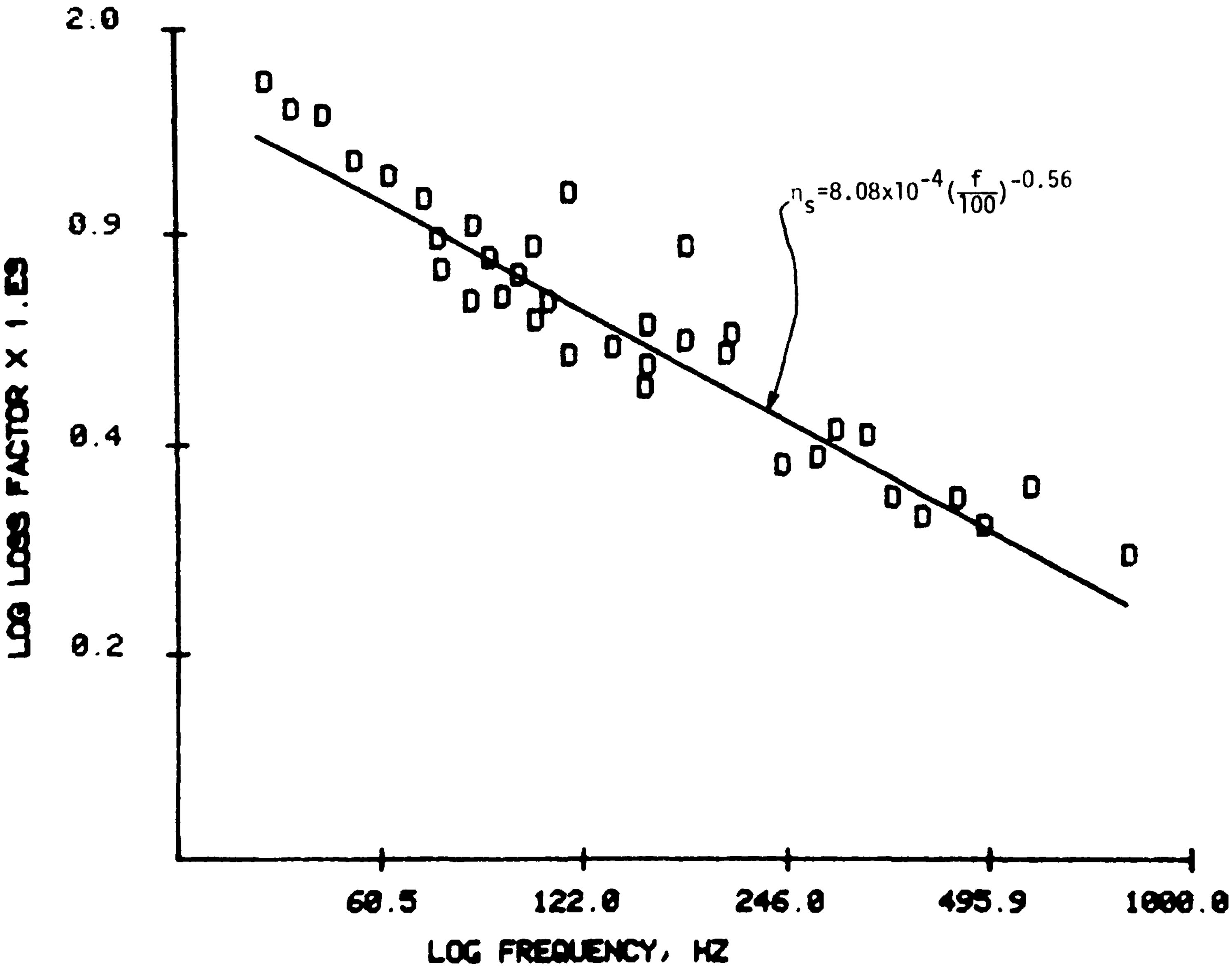

Figure 2-3. Loss Factor Dependence on Frequency for 6061-T6 Aluminum With Data Collected According to Procedures Described in Reference [19].

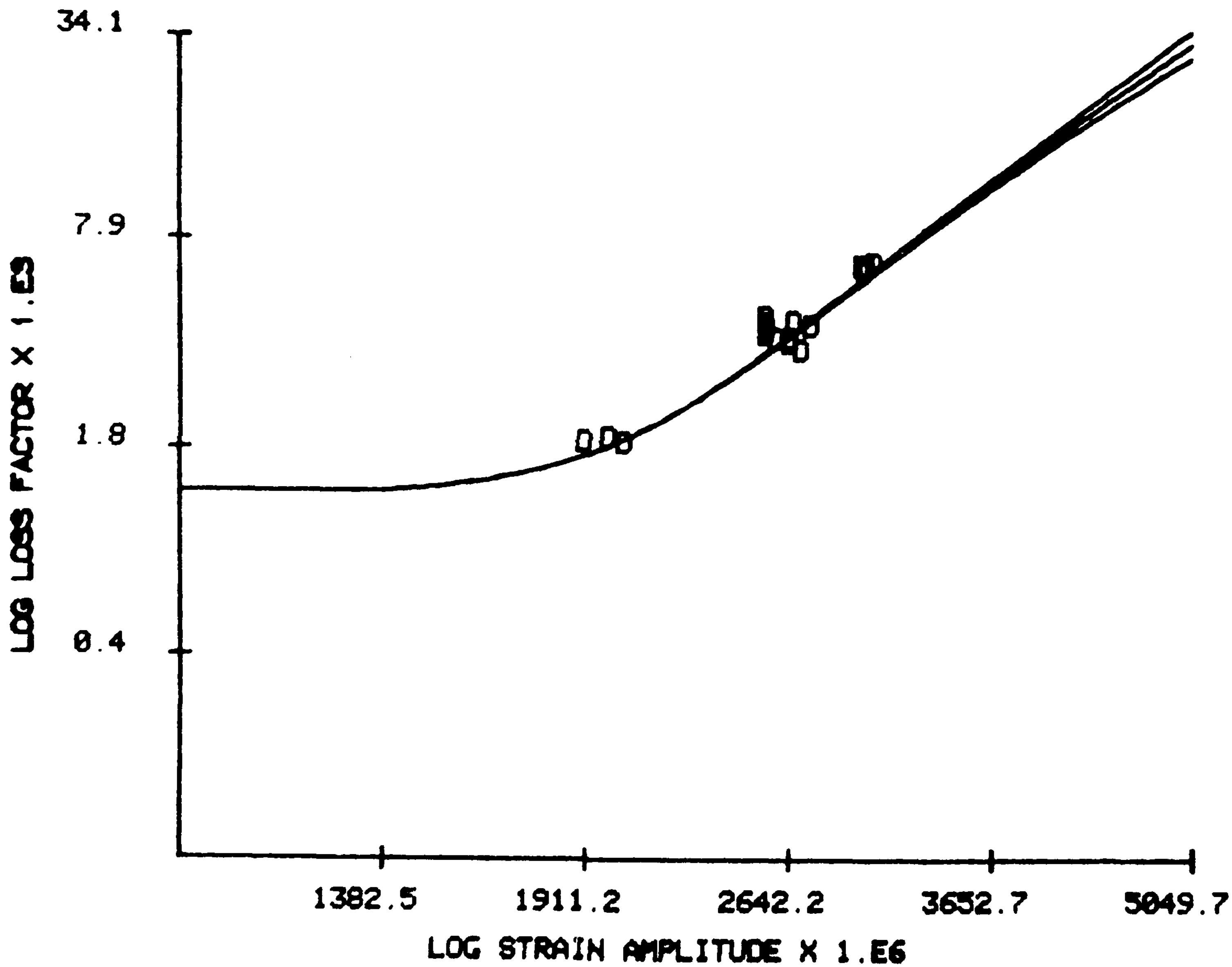

Figure 2-4. Loss Factor Dependence on Strain Amplitude for 606]-T6 Aluminum Including Confidence Intervals With Data From Reference [19].

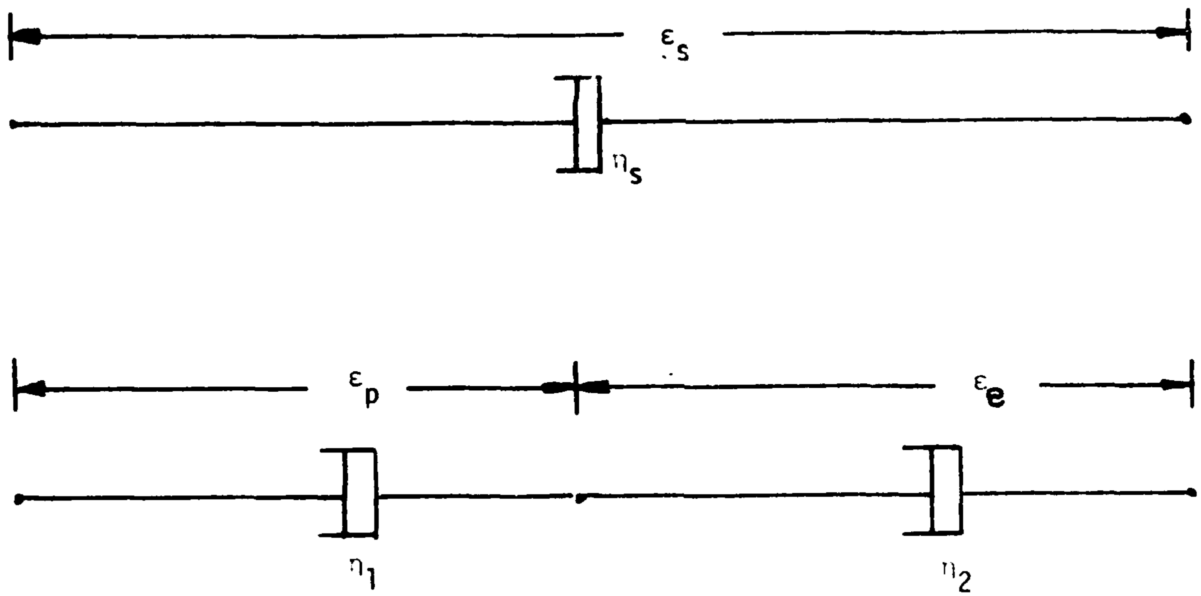

Figure 2-5. Schematic Representation of System Elements of Reversible and Irreversible Contributions to Loss Factor.

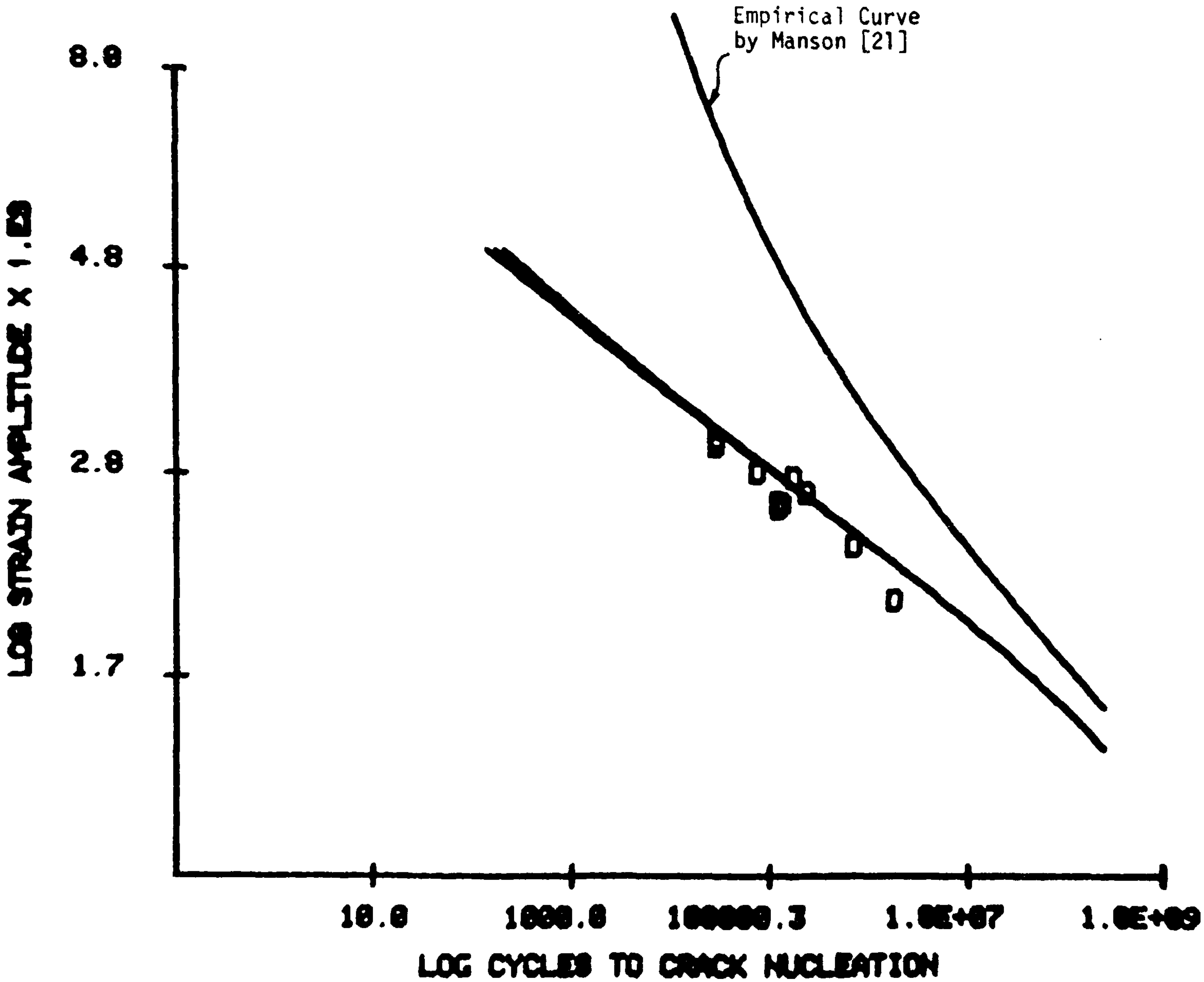

Figure 2-6. Isothermal Fatigue Crack Nucleation Lifetime Estimation Curves for 6061-T6 Aluminum Compared With Data in Vacuum According to Reference [19], Along With the Empirical Results of Reference [21].

CONTINUOUS MEASUREMENT OF MATERIAL DAMPING DURING FATIGUE TESTS

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ABSTRACT

An experimental procedure for continuously measuring strain level, temperature, and damping during resonant fatigue tests is described. The technique is based on a previous method (Gibson & Plunkett, Experimental Mechanics, Aug. 1977, pp. 297-302) using base excited cantilevered beams vibrating at resonance. The amplitude and frequency dependence of loss factor is therefore directly included in the measurement. For beams vibrating in vacuum, energy dissipation rate and temperature measurements provide a basis for thermodynamic measurements of fatigue for analysis of irreversible thermodynamics. This procedure provides a continuous measurement of energy dissipation rate during fatigue crack nucleation.

INTRODUCTION

A new hypothesis for the fundamental mechanism of *material fatigue* is presented in Reference [1]. The purpose of this paper is to describe an experimental procedure to be used in validating that hypothesis. A thermodynamic analysis of a base-excited cantilever beam vibrating at resonance reveals an entropy gain term which is a measure of the plastic (irreversible) part of the strain energy. For low strains this entropy term goes to zero, and provides a measure of the irreversibility of the thermodynamic process. The new hypothesis for fatigue failure is that this entropy gain related to the fatigue damage is a constant at fatigue failure. The plastic part of the total hysteresis energy is calculated using Whiteman's [2] probability model for yielding. Theoretical evidence from the open literature for this hypothesis is given in Reference [1], but experimental proof is also required.

The experimental proof of the entropy hypothesis for fatigue damage requires that the thermodynamic analysis given in Reference [1] be verified

in the laboratory. This means that input strain level, temperature and loss factor at the cantilever beam root must be continuously measured and compared with the theoretical predictions given in Reference [1]. The base-excited double cantilever beam has been used as a test specimen in Reference [3] for measuring material damping. However, material damping is known to increase during fatigue, so a continuous measurement is required [4]. The analog computer is capable of providing such a continuous measure of material damping during sinusoidal testing using the experimental procedure described in Reference [3]. This paper describes the details of the continuous measurement of material damping during fatigue using the analog computer. This laboratory data will then be compared with the predictions given in Reference [5] in an effort to verify the entropy hypothesis of fatigue failure.

ANALYSIS

The base excited double cantilever beam test specimen described in Reference [3] forms the basis for this experimental investigation. Although only one side is instrumented, the double beam is used for balance to prevent excessive rocking motion on the shaker head. For the test configuration of Figure 1, the equation of motion is:

$$EI \frac{\partial^4 y}{\partial x^4} + FI \frac{\partial^5 y}{\partial x^4 \partial t} + \rho A \frac{\partial^2 y}{\partial t^2} = -\rho A\ddot{Z}. \quad (1)$$

Equation (1) includes a damping term based on the viscoelastic damping model, but the data will be given in terms of the loss factor which is to be measured at a single natural frequency. The solution to equation (1) is known to be:

$$y(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t) + \ddot{Z}. \quad (2)$$

Substituting equation (2) into equation (1) and invoking the well-known orthogonality relationships the decoupled equations of motion in terms of the generalized coordinates result:

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i = Q_i\ddot{Z} \tag{3}$$

In equation (3), Q_i is the generalized force coefficient and for the cantilever beam with an accelerometer at the tip can be shown to be:

$$Q_i = -\rho A \int_0^{\ell} \phi_i(x)dx/[\rho A \int_0^{\ell} \phi_i^2(x)dx + m\phi_i^2(\ell) + J_G\phi_i'^2(\ell)]. \tag{4}$$

The mode shape is to be normalized such that $\int_0^{\ell} \phi_i^2(x)dx=\ell$. When the base is excited at the resonant frequency of the beam, $\ddot{Z} = \ddot{Z}_0 e^{j\omega_i t}$, and the loss factor can be calculated from the measured base and tip accelerations using equations (2) and (3):

$$\eta_s = 2\xi_i = \frac{Q_i\ddot{Z}_0\phi_i(\ell)}{|\ddot{y}-\ddot{Z}_0|} . \tag{5}$$

The energy dissipated per cycle is:

$$D = \int_0^{2\pi/\omega_i} \sigma\dot{\varepsilon}dt = \int_0^{2\pi/\omega_i} F\dot{\varepsilon}^2 dt = \pi E\eta_s\varepsilon_0^2 \tag{6}$$

Equations (5) and (6) can be implemented on the analog computer for continuous measurement, but require one divider and one multiplier. A strain gauge would normally be used to measure strain but strain gauges fail during fatigue and it is possible to calculate strain from measured acceleration. For a vibrating beam, the strain is:

$$\varepsilon = \frac{h}{2}\frac{\partial^2 y}{\partial x^2} = \frac{h}{2}\phi_i''(x)q_i(t) .$$

When the cantilever beam is vibrating at resonance, this is just:

$$\varepsilon = \frac{h}{2} \frac{\phi_i''(x)|\ddot{y}-\ddot{Z}|}{\omega_i^2 \phi_i(\ell)} \qquad (7)$$

Now combining equations (5), (6), and (7) and energy dissipated per cycle is:

$$D = \pi E[Q_i \phi_i(\ell) \frac{\ddot{Z}_0}{|\ddot{y}-\ddot{Z}|}][\frac{h}{2} \frac{\phi_i''(x)|\ddot{y}-\ddot{Z}|}{\omega_i^2 \phi_i(\ell)}]^2 .$$

Combining terms,

$$D = \frac{\pi E h^2}{4\omega_i^4} \frac{Q_i}{\phi_i(\ell)} [\phi_i''(x)]^2 \ddot{Z}_0 |\ddot{y}-\ddot{Z}| \qquad (8)$$

Equation (8) is a significant result since it reveals that the energy dissipated per cycle can be continuously calculated on the analog computer using just one multiplier. The mode shape function and natural frequency are well known and can be readily calculated [6]. However, the generalized force coefficient Q_i must be calculated for the particular mode shape function to be used. Although use of tabulated functions given in Reference [6] provides useful results when deflections are involved, since the strain is to be calculated a more accurate procedure is required.

Equation (8) is a more convenient form for measurement since accelerometers are easier to install and not subject to drift during fatigue as are strain gauges. The added mass of the accelerometer will effect the natural frequency and mode shape and also the measured tip acceleration.

In order to assure accurate results it is necessary to investigate the affect of the accelerometer mass attached to the tip of the beam. It is logical that the smallest possible accelerometer should be used in order to minimize that affect, but for fatigue testing, the thickness of the beam is to be varied to allow for the various strain levels tested.

When an accelerometer is attached to the beam tip as shown in Figure 2, the mode shape function $\phi_i(x)$ and natural frequency ω_i are modified and errors in strain measurements using equation (7) might become significant.

Figure 2 shows a free body diagram of the accelerometer attached to the beam tip. The mass and rotary inertia of the accelerometer introduce a dynamic boundary condition at the end of the beam which results in the following equations for the dynamic shear and moment.

$$EI \frac{\partial^2 y}{\partial x^2} = J_G \frac{\partial^3 y}{\partial x \partial t^2} + \frac{m}{4}(e^2+d^2)\frac{\partial^3 y}{\partial x \partial t^2} + \frac{md}{2}\frac{\partial^2 y}{\partial t^2} \quad (9)$$

$$EI \frac{\partial^3 y}{\partial x^3} = m \frac{\partial^2 y}{\partial t^2} + \frac{md}{2}\frac{\partial^3 y}{\partial x \partial t^2} - \frac{me}{2}\left(\frac{\partial^2 y}{\partial x \partial t}\right)^2 \quad (10)$$

Equation (10) contains a nonlinear term which vanishes when $e = 0$, indicating that if the accelerometer center of mass lies on the beam neutral axis, the response will be linear. By assuming that the response is sinusoidal in order to solve for natural frequencies, it is possible to show that the nonlinear term in equation (10) introduces an additional frequency in the response. Based on numerous laboratory measurements, it is apparent that the nonlinear term does not substantially effect the natural frequency for moderate tip amplitudes so is neglected.

The modal deflection function is well-known [6], and applying the boundary conditions at the root the following form results.

$$\phi_i(x) = B[\cos k_i x - \cosh k_i x + R_i(\sin k_i x - \sinh k_i x)]. \quad (11)$$

In equation (11), R_i is a mode shape constant and B is the modal deflection amplitude indicated in Figure 3. Equations (9) and (10) are combined with (11), and the following transcendental algebraic equations result:

$$R_i = - \frac{\cos\lambda_i(1-\mu\nu_3\lambda_i^2) + \cosh\lambda_i(1+\mu\nu_3\lambda_i^3) + \mu\lambda_i^3(\nu_1+\nu_2+\nu_3)(\sin\lambda_i+\sinh\lambda_i)}{\sin\lambda_i(1-\mu\nu_3\lambda_i^2) + \sinh\lambda_i(1+\mu\nu_3\lambda_i^2) - \mu\lambda_i^3(\nu_1+\nu_2+\nu_3)(\cos\lambda_i-\cosh\lambda_i)} \tag{12}$$

$$\begin{aligned} &- \sin\lambda_i + \sinh\lambda_i - \mu\lambda_i(\cos\lambda_i - \cosh\lambda_i) + \mu\nu_2\lambda_i^2(\sin\lambda_i + \sinh\lambda_i) \\ &+ R_i[\cos\lambda_i + \cosh\lambda_i - \mu\lambda_i(\sin\lambda_i - \sinh\lambda_i) - \mu\nu_2\lambda_i^2(\cos\lambda_i - \cosh\lambda_i)] = 0. \end{aligned} \tag{13}$$

When $\mu=0$, equations (12) and (13) reduce to the familiar cantilever beam solutions. When the mass of the accelerometer is included the natural frequency is decreased, and the mode shape function is modified. However, equations (12) and (13) only indicate a part of the error incurred from using equation (7) to measure strain. Referring to Figure 3, the actual strain is:

$$\varepsilon = \frac{h}{2} \frac{\phi_i''(o)B}{\phi_i(\ell)} ,$$

since B is the theoretical beam deflection. The accelerometer only measures the perpendicular acceleration, given by:

$$A_G = \left\{ \left(\frac{\partial^2 y}{\partial t^2} + \frac{d}{2} \frac{\partial^3 y}{\partial x \partial t^2}\right) - \frac{e}{2} \left(\frac{\partial^2 y}{\partial x \partial t}\right)^2 \right\} \frac{1}{\cos\theta} \tag{14}$$

The acceleration can be reduced to:

$$A_G = \phi_i \left\{ 1 + \nu_3 \frac{\bar{\phi}_i'}{\phi_i} - \nu_2 \left(\frac{\bar{\phi}_i'}{\phi_i}\right)^2 \frac{B}{\cos\theta} \right\} \frac{\omega_i^2 B}{\cos^2\theta} \tag{15}$$

Then the error in strain measurement is:

$$E = \frac{\dfrac{h\phi_i''(o)A_G}{2\omega_i^2\phi_i(\ell)} - \dfrac{h\phi_i''(o)B}{2\phi_i(\ell)}}{\dfrac{h\phi_i''(o)B}{2\phi_i(\ell)}}$$

In dimensionless terms,

$$E = \left\{ 1 + \nu_3 \frac{\bar{\phi}_i'}{\phi_i} - \nu_2 \left(\frac{\bar{\phi}_i'}{\phi_i} \right)^2 \frac{B}{\cos\theta} \right\} \frac{1}{\cos^2\theta} - 1 \tag{16}$$

Equation (16) is plotted versus strain in Figure 4 for various sizes of beams and accelerometers. As strain increases the error increases since θ increases. In laboratory tests this error was found to be less than the error in reading the voltmeter. Figure 4 shows that for strains less than about 3000 μs, which covers the range of interest in high cycle fatigue life studies, the error in the strain measurement using an accelerometer is less than 5%.

Equation (15) can be readily used to measure strain from accelerometer readings. Equation (7) is the basic strain-accelerometer relationship and strain calculated from A is just:

$$\varepsilon(o) = \frac{h\phi_i''(o)\cos^2\theta A_G}{2\omega_i^2\phi_i(\ell)\left[1 + \nu_3 \frac{\bar{\phi}_i'}{\phi_i} - \nu_2\left(\frac{\bar{\phi}_i}{\phi_i}\right)^2 \frac{B}{\cos\theta}\right]} \,. \tag{17}$$

Figure 4 shows that the strain measurement using equation (7) is within about 5% for the strains of interest here. Of course, the error is worse if a larger accelerometer is used but is still less than about 10% even at strains above the yield point of aluminum. When the analog computer scale factor is introduced, the measured quantity is related to the theoretical quantity according to:

$$S_D D_V = D.$$

Even though the strain measurement is accurate, the damping measurement at high strains includes a significant air damping term [3]. Therefore, in order to accurately simulate the thermodynamic analysis presented in

References [1] and [5], it is necessary to enclose the test specimen in a plexiglass vacuum chamber as described in Reference [3]. In this way, the energy dissipation measurement will not be corrupted by the presence of air damping.

The analog computer scale factor can be calculated from Figure 1 and equation (8):

$$S_D = \frac{\pi E h^2 Q_i [\phi_i''(o)]^2}{4\omega_i^4 \phi_i(\ell)} \frac{100 C_Z C_Y}{G_Z G_Y G_D} \tag{18}$$

Since equation (17) contains the generalized force coefficient, the calculation indicated in equation (4) must be carried out.

Equations (1) - (18) have been implemented in measuring strain, temperature, and energy dissipation rate versus time for cantilever beam specimens failed in fatigue by vibrating at resonance. Typical results of that experimental effort are described in the next section.

EXPERIMENTAL PROCEDURE

Double base-excited cantilever beams were vibrated at resonance in a vacuum while measuring temperature, strain level, and damping. Figure 5 is a photograph of typical beam specimens including fracture which always occurs at the base. Cracks are present at about a one percent decrease in the natural frequency. The beam specimens are 6061-T6 Aluminum. A plot of loss factor versus strain level is indicated in Figure 6 along with data[1] [5]. Figure 7 is a photograph of a beam specimen mounted inside the plexiglass vacuum chamber.

The data are recorded on a four-channel strip-chart recorder. Measurements of strain using the accelerometer were compared with measurements of strain using strain gauges with favorable results except that

the strain gauges always failed before the fatique test was complete. Although the gauges could be moved farther out on the beam to decrease the strain level and therefore increase the life, the accelerometers proved to be a superior instrumentation package. A pressurized bulkhead connector is indicated in Figure 7 for the electrical connections.

Figure 8 is a photograph of a typical data record. Since the temperature increase was small it is omitted. The frequency must be carefully adjusted to beam resonance which corresponds to the minimum base acceleration, as in Reference [3]. This adjustment is routine at the beginning of the test, but as fatigue progresses the natural frequency decreases due to the modulus change and resonance must be readjusted. This procedure can be readily identified in the data of Figure 8. As fatigue progresses, the energy dissipation rate increases slightly until it becomes impractical to maintain resonance as shown on the last part of the data record in Figure 8. This part of fatigue is crack propogation and can be carried out by constantly changing the frequency to follow the modulus change. This is impractical, however, as it is necessary to maintain changing resonance while maintaining constant strain amplitude. Since the measurement theory given in the previous section only holds in the absence of cracks, the tests were terminated after a one percent change in natural frequency. The fatigue lifetime was measured by a time-code generator on the strip chart recorder.

DISCUSSION OF RESULTS

This procedure is a valid one as long as the modulus does not change substantially. For studies of fatigue crack initiation in unblemished metals the results are good. However, when cracks are present this measurement procedure fails. When the modulus change is more rapid, it is possible to excite the specimens with narrow-band random noise and

accomplish this continuous measurement procedure using FFT analysis and a microcomputer [7].

When the excitation strain level is much lower than the yield point of the material, this procedure is an accurate way to continuously measure the energy dissipated during fatigue and thus collect thermodynamic data for use in irreversible thermodynamic analysis of fatigue. The base excited cantilever beam is a convenient one for fatigue studies since the maximum strain is at the base and cracks always form at the base.

Measurement of strain level on base-excited cantilever beams with accelerometers is an accurate and reliable as well as convenient procedure. Installation of accelerometers is much easier than strain gauges, there is no bridge to balance, and accelerometers do not fail during fatigue. Accuracy is on the order of one percent or less as compared to strain gauges for most beam thicknesses of practical significance. The fatigue crack always forms at the base of the cantilever beam where the strain is maximum, giving a predictable way to "pre-crack" specimens for crack propogation studies.

CONCLUSIONS

An experimental procedure for continuously measuring strain level, temperature, and energy dissipation rate during fatigue tests is described. Energy dissipation rate and temperature are continuously measured providing a basis for thermodynamic analysis when testing in vacuum where there is negligible convection heat transfer. For moderate strain levels and for materials which do not show a rapid modulus change the procedure gives satisfactory results up to crack initiation. In the presence of cracks a fracture mechanics analysis is necessary, but using narrow-band

random excitation allows this procedure to be extended to the crack propogation study. That research is being planned at this writing.

ACKNOWLEDGEMENTS

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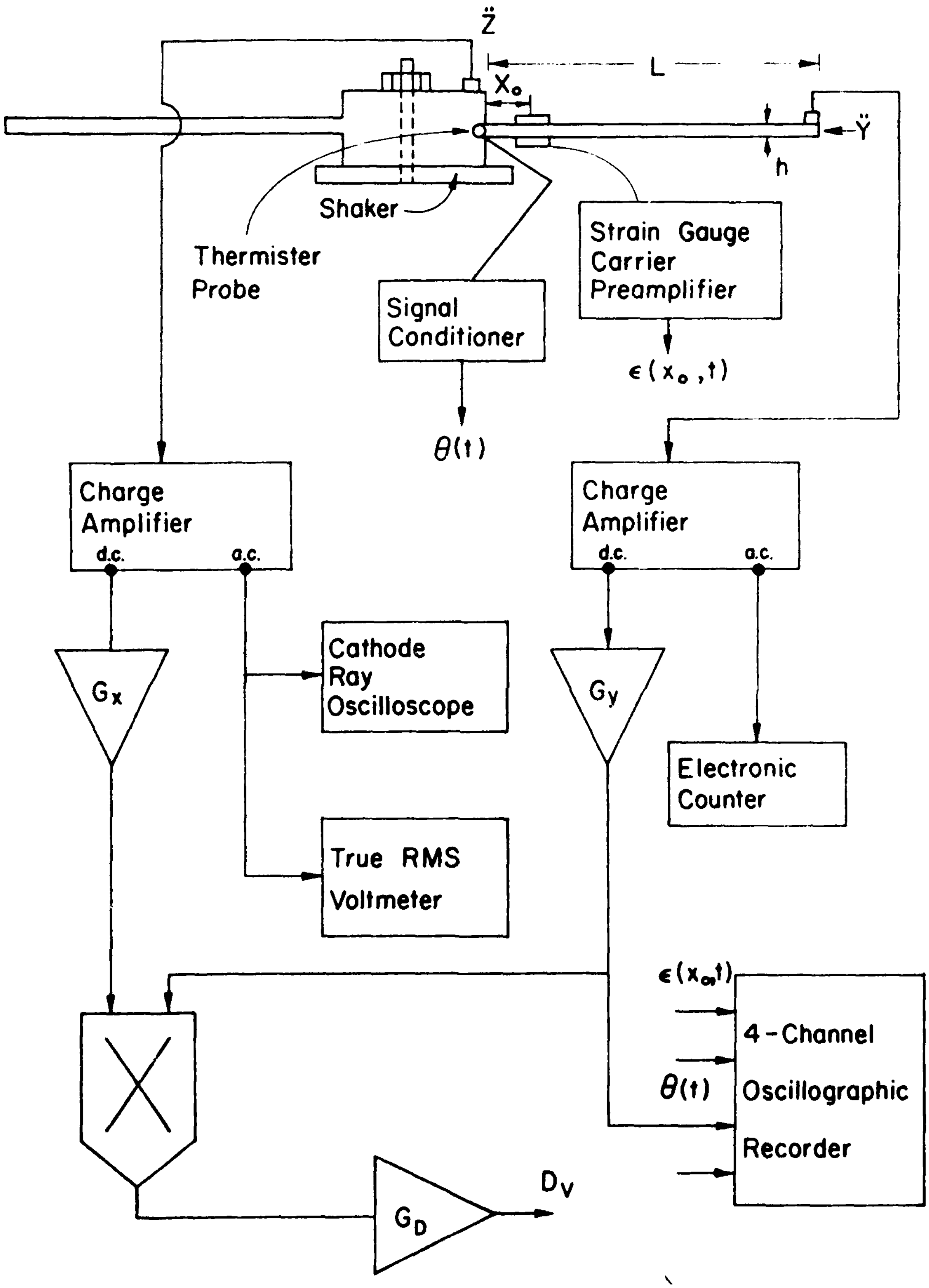

Figure 3-1. Experimental Apparatus.

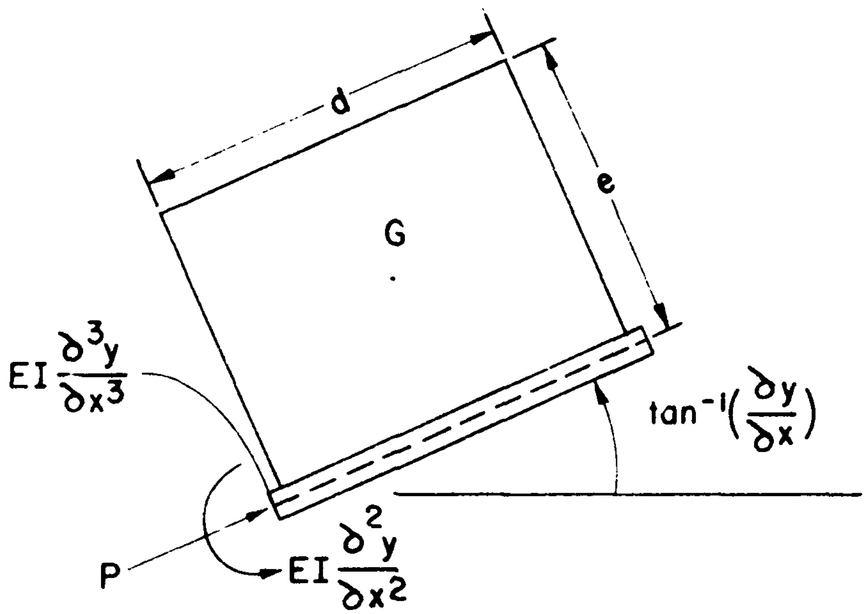

Figure 3-2. Free-Body Diagram of Accelerometer Attached to Beam Tip.

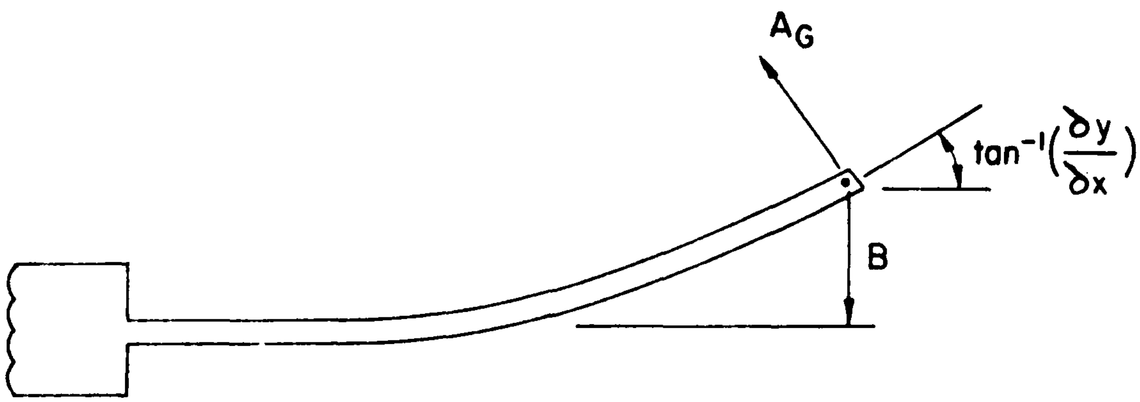

Figure 3-3. Geometry of the Measured Tip Deflection.

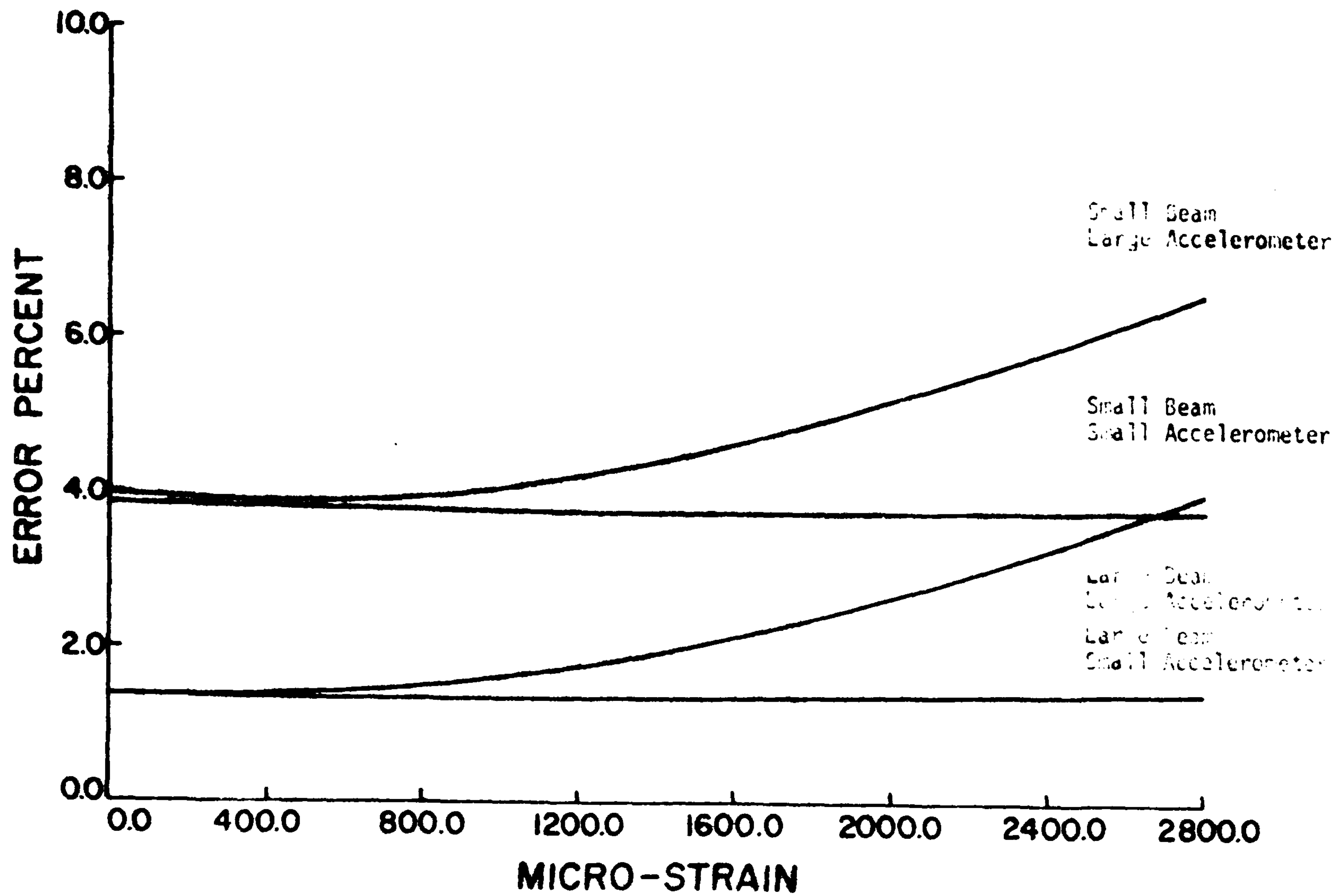

Figure 4. Typical Error in Strain Measurement Using Accelerometers.

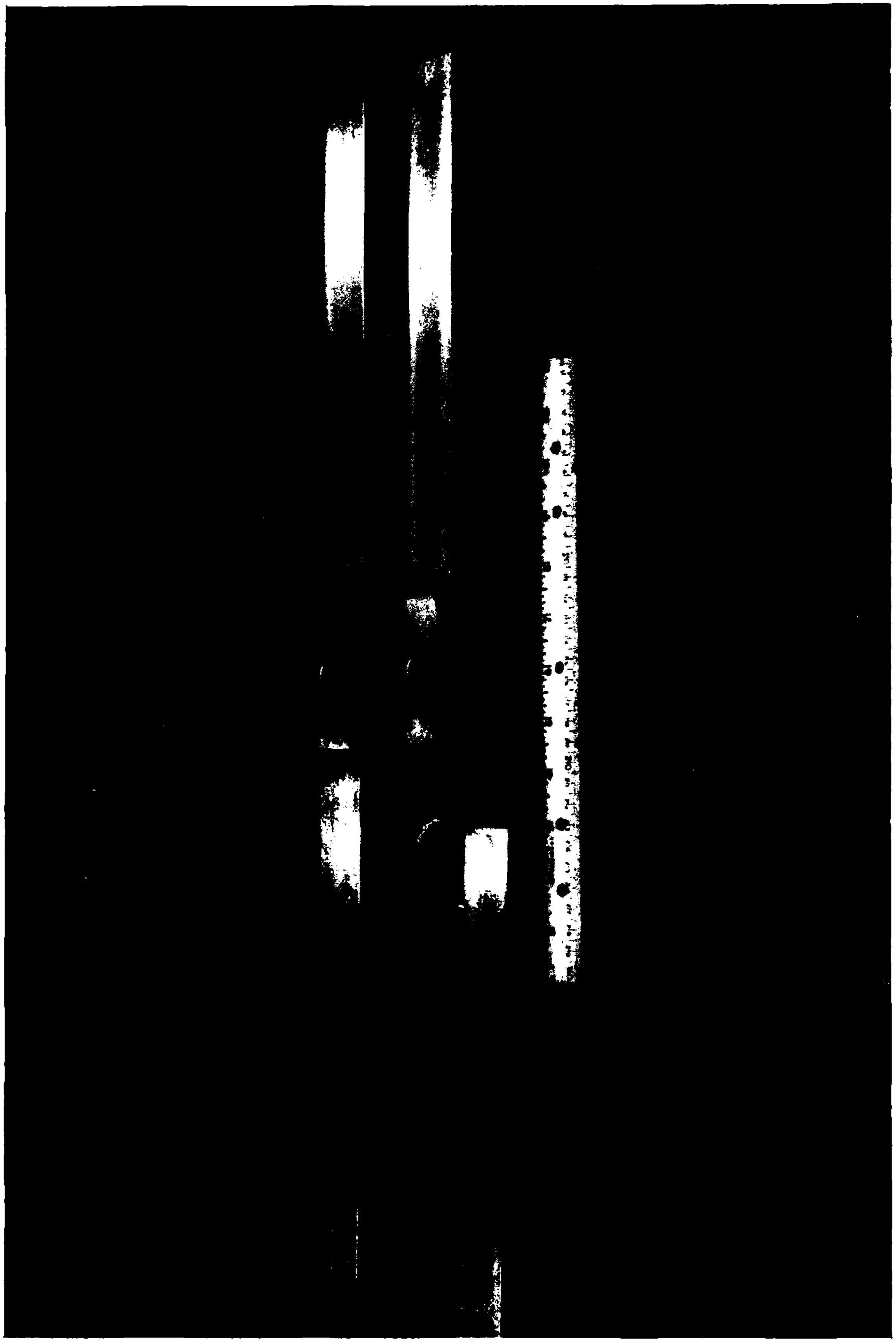

Figure 3-5. Typical Beam Specimens.

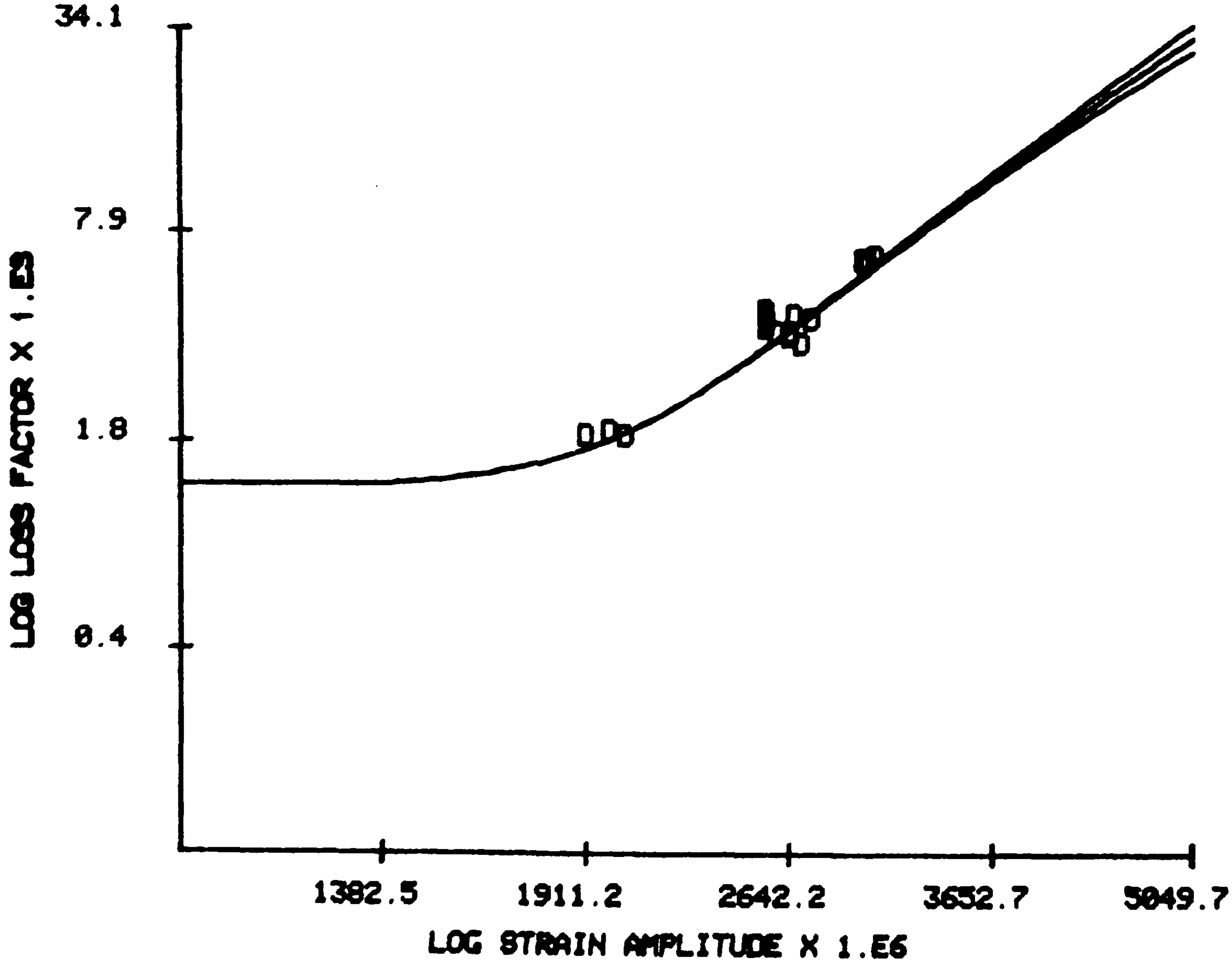

Figure 3-6. Loss Factor versus Strain Level for 6061-T6 Aluminum

Figure 3-7. Beam Specimen Mounted Inside a Vacuum Chamber.

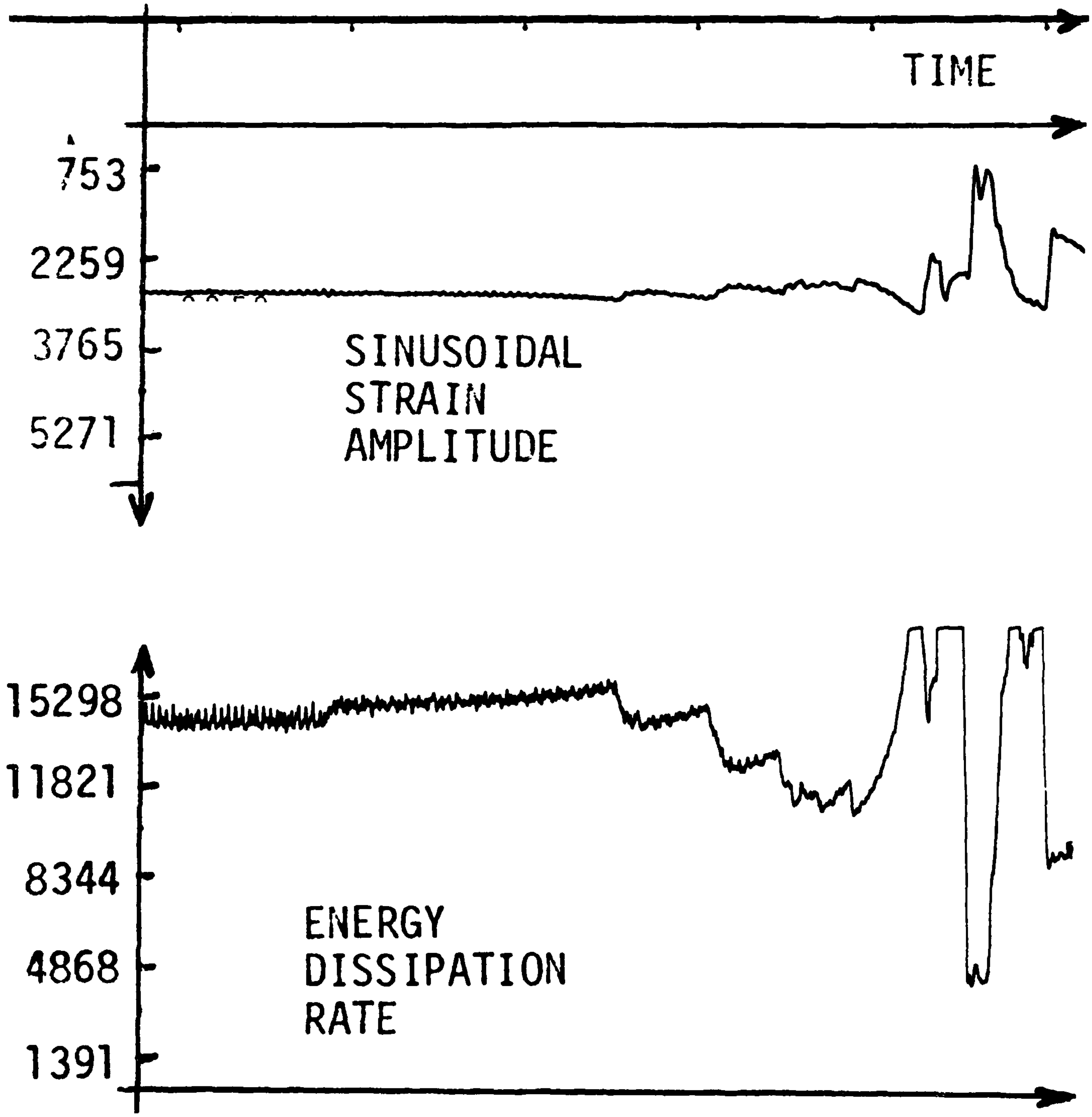

Figure 3-8. Typical Thermodynamic Fatigue Data During Crack Initiation: Natural Frequency Changed from 40.8 Hz to 39.3 Hz.

AD-A134 767 ENTROPY PRODUCTION DURING FATIGUE AS A CRITERION FOR FAILURE THE CRITICAL. (U) NEBRASKA UNIV LINCOLN COLL OF ENGINEERING AND TECHNOLOGY P W WHALEY 15 AUG 83 2/2

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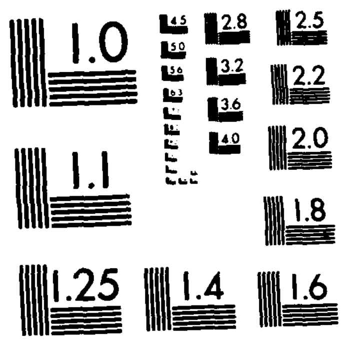

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